# PRIOR KNOWLEDGE TO BUILD THE MATHEMATICAL REPRESENTATION ABILITY IN GEOMETRY

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## Abstract

Representation ability is one of the most important abilities that must be had by students to process the information, answer the questions, and solve the problems. Prior knowledge becomes the most crucial thing that makes students able to connect all of the exist information that they have to build new knowledge. The purpose of this study was to analyze the missed prior knowledge that led the students had difficulties, in understanding and answering the questions, about geometry. The study involved one instrument of 2 questions of geometry and it was given to 19 students of $12^{th}$ grade students that were chosen randomly in a senior high school in Dolok Masihul, North Sumatera without any intervention on learning process in the class. The questions given was adjusted to the operational form of mathematical representation ability which have the Cognitive level 4 ($C_4$) for question number 1 and $C_5$ for question number 2 based on Bloom's taxonomy. This study showed that prior knowledge become an essential thing to build the students' representation ability to make new knowledge, especially in geometry. The most trouble topics which make them diffficult to understand the question are ratio, line and angles, power and square operation, and the last rectangular and triangle. Reflecting to the result, it is better for teacher to make sure that students have enough prior knowledge to make them easier to build new knowledge and make a fun and meaningful learning process in order to make information saved well in student's long term memory.

**Keywords**: Mathematical representation ability, geometry, prior knowledge, learning obstacles

## 1. INTRODUCTION

Based on Nasional Exam (Ujian Nasional-UN) 2015 result in Indonesia from the Ministry of Education and Culture in Indonesia (Kementerian Pendidikan dan Kebudayaan, 2017), mathematics has the lowest score mean from 3 classification of senior high school in Indonesia. They are 59.17 from 758,067 students of science class in senior high school, 55.76 from 852.878 students of social class in senior high school and 48.24 from 1,241,348 students from vocational high school (maximal score is 100). It seems like mathematics still become the most difficult lesson from senior high school in Indonesia. It becomes a problem because mathematics are the foundation and the framework to face the globalization. In education, mathematics becomes a tool to develop student's logical thinking, accuracy, spatial awareness, and gives effort to solve challenging problems from the development of mathematical abilities (Goldin, 2014, p.4). The National Council of Teachers of Mathematics (NCTM, 2000, p.4) said that mathematical abilities have principles and process standards, they are problem solving, reasoning and proof, communication, connections, and representations.

This study would focus on the representation ability. Representation is the way as fundamental in people can understand mathematical ideas. Students use representation as tools to support their mathematical understanding by constructing abstract idea into concrete idea by using logical thinking. Representation is a sign or a configuration of signs, characters, or objects which mark and configurate to represent, describe other than itself (Goldin, 2014, p.4), so it will support students in learning, especially for communicating and connecting concept to solve the mathematical problem. From the explanation, students which has low ability of representation will show a lack of skilled in generating ideas, asking questions and responding the questions or opinions of others. Because of that, there is a bidirectional relation between representation and cognitive abilities (Stievenart, Roskam, Meunier & Moortele, 2011, p.64). There are five kinds of representation that are useful for mathematical understanding: (a) real life experiences, (b) manipulative models, (c) pictures or diagrams, (d) spoken words, and (e) written symbols. The use of visual representation is a highly recommended instructional practice in mathematics (Garderen, Scheuermann, Poch & Murray, 2016, p.1) and representations can be used to teach word problem solving effectively (Jitendra, Nelson, Pulles, and Houseworth J, 2016, p.22). Good representation ability will came from good prior knowledge of students.

In the context of learning, prior knowledge can be defined as the initial ability of a learner that can be a starting point to see how much student's behavior changes after he/she follows the learning process. Prior knowledge greatly influences the learning process (Matsuda, et.al. 2013, p. 1154); affects 81 students from two different classes who were given different treatment in the form of a game-based learning system (Chen and Huang, 2013, p.177); and affects the interaction with learning systems and demonstration abilities (Bringual et al, 2016, p.2). From the explanation, it can be concluded that prior knowledge is one of the most important thing to build new information in student's mind. Also become some points that must be connected to make representation from the information given.

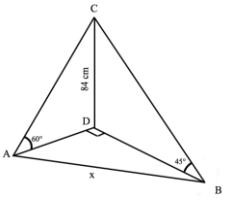
Geometry as an indispensable topic in mathematics, it is considered to be a rich area to foster student problem solving and reasoning skills (Jupri, 2017, p.1). To solve the problem, students need to use their representation of relevant information (Krawec, 2014, p.103). Relevant information comes from the information of the question given that connected to student's prior knowledge, so students able to make visual image that will be used to imagine the purpose of the question given. The study would to analyze which prior knowledge that missed by students so they have difficulty in answering the questions given about geometry.

The study uses the questions that categorized on the cognitive levels of Bloom's Taxonomy which categprized into remembering ($C_1$): retrieving, recognizing, and recalling relevant knowledge from long-term memory; understanding ($C_2$): constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining; applying ($C_3$): carrying out or using a procedure through executing, or implementing; analyzing ($C_4$): breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing; evaluating ($C_5$): making judgments based on criteria and standards through checking and critiquing and creating ($C_6$): putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing (Anderson & Krathwohl, 2001, pp. 67-68).

## 2. METHODOLOGY

19 students of 12th grade students that were chosen randomly in a senior high school in Dolok Masihul, North

Table 1. Question of geometry that used in the research

| No | Aspect of Mathematical Representation Ability | Cognitive (C) | Questions |
|---|---|---|---|
| 1 | Mathematics expression (formulas) | $C_4$ |   Given a block has a ratio of edges = 3: 6: 2. The length diagonal space is 21 cm. What is the volume of the block? |
| 2 | Words | $C_5$ | Satisfy x = $56\sqrt{3}$ cm and make the suitable daily situation to represent the figure beside. |

Sumatera were given 2 questions; operational form of mathematical representation ability which have the Cognitive level 4 ($C_4$) for question number 1 and $C_5$ for question number 2 based on Bloom's taxonomy as shown in Table 1. Students received no specialization intervention on learning process in class. The questions given was used to analyze how far they able to answer it. Questions also had been validated by mathematics teacher in a senior high school and a mathematics lecture in Universitas Pendidikan Indonesia.

After the questions were answered by students, it would be corrected to find some problems about students' understanding of mathematics concepts as its prior knowledge that made them difficult to answer it. Then student's score were be sorted from highest to lowest to determine who would be interviewed and discussed in this study.

## 3. RESULT AND DISCUSSION

From the document of Kurikulum 2013 as national educational curriculum after revision in 2016, there are 5 main aspects that will be learned by students in senior high school. They are algebra and trigonometry (10th grade); algebra and calculus (11th grade); geometry and measurement; statistics and probability (12th grade). Geometry will be learned at 12th grade especially for three dimensional geometry. That's why researcher take data from 12th grade for this research. After questions were given, researcher did correction of their answer and analyzed it. Researcher found some interesting facts that will be used for education, especially for mathematics teacher who teaches geometry topic.

### 1.1 Question Number 1

One of the expected solution for question number 1 is making the known and asked information before doing next steps which drawing the block and putting all of the information given on the question. Finding the solution by determining x by using information about its space diagonal and calculating AC and AG by using Pythagoras so we will find the value of x. After that, using the ratio of its length, height and width to determine its volume. Fig. 1 shows 3 examples of students' answer sheet which researcher chosen from 3 categories after did correction and sorted the score from highest to lowest. Fig. 1A shows the answer from the lowest score. Fig. 1B from the medium score and Fig. 1C from the highest score.

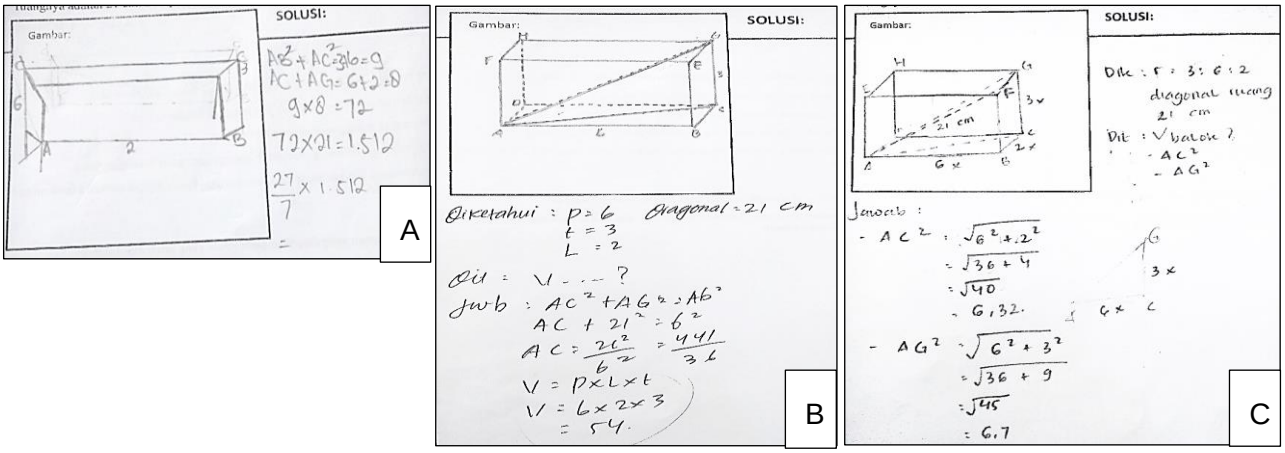



Fig. 1. Example of student's solution for question number 1

Student A in Fig. A confused to draw the figure of cubes by many unnecessary lines she made and connected the lines not in the corresponding point. She also couldn't classify the known and asked information. It made they difficult to image how to solve the problem. Besides that, she confused in using the concept of ratio; algebra operations; line and angles; and Pythagoras. Suddenly she wrote $AB^2 + BC^2 = 3 + 6 = 9$. In the other notation from the equation she thought that $AB = \sqrt{3}$ and $BC = \sqrt{6}$. It is not suitable with her own cube figure that $AB = 2 \neq \sqrt{3}$ and her $BC$ is a plane diagonal which $\neq \sqrt{6}$. Based on her own cube figure again, 6 is length of C into a point without name there. Then she wrote that $AC + AG = 6 + 2 = 8$ without any information in her answer sheet why she thought like that. To make it sure, researcher did interview to students who discuss in this paper. From the conversation, it could be understood that she answered the question by just seeing the information about edges ratio and diagonal length of the block she made, even didn't understand about the figure and formulas. That's why she didn't have enough prior knowledge and couldn't construct her representation ability to answer this.

Student B able to draw the block figure and the components based on the question, made known and asked information even he didn't care about the ratio and didn't understand about Pythagoras concept. It shown from $AC^2 + AG^2 = AG^2 \Leftrightarrow AC + 21^2 = 6^2$ when it should be $AC^2 = AG^2 - GC^2$ and he made $AG = 21 = 6$ shown that he confused to determine about the length of line. Beside that he also had trouble in doing algebra operation from $AC = \frac{21^2}{6^2}$ which should be $AC = 6^2 - 21^2$ if we continue his steps. Actually he known the volume of the block. But he thought that $3x = 3$. In fact it is different. He also been interviewed and could be analyzed that actually he known that diagonal is the key to answer the question. But he didn't know how to use it.

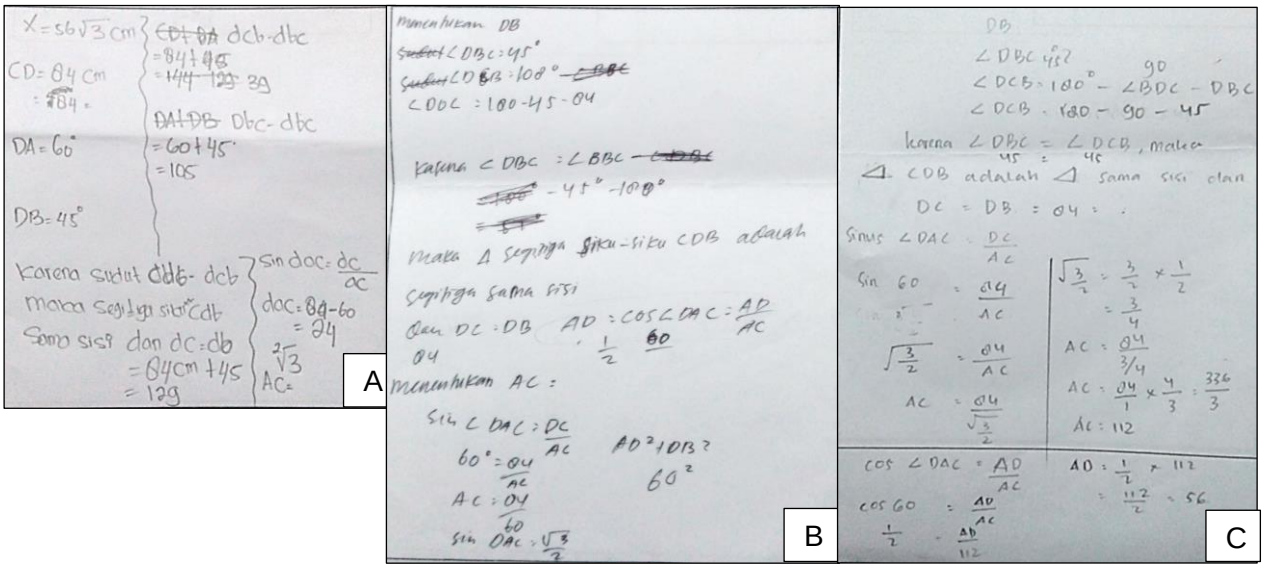
Student C able to drew the block figure and its components based on the question by writing the ratio; known and asked information. But while calculated, she didn't use the ratio. She understood enough about the Pythagoras, algebra operation, power and square operation. From the conversation, student C had difficulties in understanding the question and remembered the steps even she had good prior knowledge. It made she couldn't complete the answer but able to relate each information given. Over all, the students' problem inform in the Table 2. Only 70% students able to make the block figure and its component based on the information given, 20% students able to make known and asked information and 50% students understand the head steps to answer it. But mostly students difficult in using the concept of mathematics to complete the answer.

**Table 2.** Percentation of student's difficulties in answering geometrical question number 1, categorized by topics with its grade level at school by using Kurikulum 2013 in Indonesia

| Students' ability | Making block Figure and its component based on information given | Making known and asked information | Understanding Head Steps to Answer the Question | Mathematics Topics and Grade Level to Learn it | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | 7th grade | | | 8th grade | 9th grade |
| | | | | Ratio | Algebra operations | Line and angles | Phytagoras Theorem | Power and squre operations |
| Able to solve and correct | 70% | 20% | 50% | 10% | 70% | 20% | 20% | 5% |
| Able to solve but incorrect | 30% | 25% | 40% | 70% | 15% | 65% | 55% | 30% |
| Disable | 0% | 45% | 10% | 20% | 15% | 15% | 25% | 65% |

## 1.2 Question Number 2

Qestion number 2 is about a pyramid. It was given to make suitable daily situation based on the figure given, i.e. the situation on the sea. First of all, students must complete their necessary information by determining length of AB, DB, AC and AD by using concept of trigonometry; especially sine and cosine. Then




determining AB by using Pythagoras.

Fig. 2. Example of student's solution for question number 2

Researcher took the answer sheets from same students with the sheets which discussed for question number 1 above as in Fig.2. Fig. 2A shows the answer from the lowest score. Figure 2B from the medium score and Fig. 2C from the highest score. Student A tried to recall the information given into her answer sheet. She took $x = 56\sqrt{3}\,cm$ as known information, even actually it is the length which should be satisfied. It was correct while $CD = 84\,cm$. But became strange while she wrote $DA = 60°$ and $DB = 45°$. $DA$ is a representation as a line and $60°$ is an angle. Both of it are different but she made it same. So did $= 45°$, $dcb = 84$ . It showed that she didn't understand about the concept of line and angles. Then she subtracted $dcb - dbc = 84 - 45$ without any information how could it be. Then, "because the angle $adb - dcb$, then the right triangle CDB is an equilateral triangle which $dc = db = 84\,cm + 45 = 129$". It shown that student A also not so understand about the concept of triangle. She known the formula of sine but didn't understand how to use it. From the conversation, it was true that she didn't understand about the concept of angle. She just use the information given in the figure without made the relation between each information to make new information.

Student B started to answer the question by recalling the information into his answer sheet. He also didn't understand about the angle. It could be seen from the written $\angle DOC = 180 - 45 - 84$. There is no information about $\angle DOC$ in the figure and he also ignore the different between angle and line. 84 is length of DC. He known the formula of sine, but couldn't do calculation by using the angle from $\sin\angle DAC = \frac{DC}{AC} \Leftrightarrow 60° = \frac{84}{AC} \Leftrightarrow AC = \frac{84}{60}$. Suddenly $60°$ changed into $60$, he made it same. From the interview result it was clear that student B didn't understand about the angle.

Student C also recall the information in her answer sheet. She was better to calculate by using angle than 2 students before and able to conclude that $\measuredangle CDB$ was a right triangle from $\angle DBC = \angle DCB = 45$. Even she didn't put symbol ° as a sign for an angle, she able explained her answer well. She also understood about the formula of sine and cosine. She just didn't careful in did calculation. In the interview result, she told me that she must think it carefully and lack of time to answer this question. She answered systematically based on her logic and the formula she knew. All of the students enable to represent the figure into word which suitable with daily situation. Because they don't get all of the necessary information yet.

**Table 3.** Percentation of student's difficulties in answering geometrical question number 2, categorized by topics with its grade level at school by using Kurikulum 2013 in Indonesia

| Students' ability | Representing visual become word | Making known and asked information | Understanding Head Steps to Answer the Question | Mathematics Topics and Grade Level to Learn it | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 7th grade | | | 8th grade | 9th grade | 10th grade |
| | | | | Line and angles | Rectangular and triangle | Algebra operations | Phytagoras Theorem | Power and squre operations | Trigonometry |
| Able to solve and correct | 100% | 15% | 75% | 20% | 30% | 45% | 40% | 20% | 65% |
| Able to solve but incorrect | 0% | 5% | 25% | 75% | 60% | 50% | 15% | 50% | 20% |
| Disable | 0% | 80% | 0% | 5% | 10% | 5% | 45% | 30% | 15% |

Over all the students' problem for question number 2 is inform in the Table 3. 100% students enable to represent visual become word because they couldn't complete the necessary information. Even 75% students understood head steps to answer the question, they still couldn't complete it yet because lack of the prior knowledge which only 20% students knew the concept of line and angles, 30% students understood about rectangular and triangle, 20% students understood about power and square root operation well, 45% students able to do algebraic operation here, 40% knew about the Pythagoras formula and 65% students knew about the trigonometry formula. But it wouldn't work while they didn't understand about the concept of line and angels.

Another problem is they have less prior knowledge for that. Actually the mathematics concepts have been learned by them in junior high school and actually mathematics characteristic in learning is continuously. So they should use it in the topic before in $10^{th}$ and $11^{th}$ grade. But in fact they are not understand it yet. Even in the basic topic like algebra operation; power and square operation; and another geometrical concept such as ratio; line and angles; and Pythagoras theorem.

This study also did interview the mathematics teacher and take the conclusion that trigonometry is the most difficult topic to teach. For geometry topic, he said that students very difficult in making image the position of

point, line and plane in the space to relate the information given in question and know what the question is. May be they know the point position, but enable to process it become a solution. It also can be happened because they don't have enough prior knowledge as the connected information to represent the information given in their mind and don't know what to do to answer the question.

## 4. CONCLUSION

In answering the geometry questions, students still difficult in making block figure and its component based on the information given as the main thing to answer the questions. They also have trouble in determining known and asked information to represent the question in word form. Even they understand the head steps to answer the questions, they don't have prior knowledge enough about the topics of ratio; algebra operations; line and angles; phytagoras theorem; power and square operations; rectangular and rtiangle; and trigonometry. The most trouble topics which make them diffficult to understand the questions are ratio; line and angles; power and square operation; rectangular and triangle.

From the explanation, it is better for teacher to make sure students have enough prior knowledge to make them easier in building new knowledge, make a fun and meaningful learning process in order to make information saved well in student's long term memory. For next researcher is how to build up their prior knowledge that can support geometry learning that suitable with the time given in learning process.

## 5. ACKNOWLEDGEMENT

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