AN ANALYSIS OF STUDENTS' METACOGNITIVE SKILLS IN SOLVING LINEAR EQUATION PROBLEMS

Cintya Putri Permata[1], Sufyani Prabawanto[2]

[1]Universitas Pendidikan Indonesia, Indonesia, cintyaputripermata@upi.edu
[2]Dr., Universitas Pendidikan Indonesia, Indonesia, sufyani@upi.edu

Abstract

One of the mathematical abilities that focus on the ability to think about something that has been thought of is metacognitive ability. Metacognitive ability consists of knowledge, skills, and concepts. The purpose of this study was to describe the metacognitive skills of students and to know the type of difficulties in solving the problem of linear equations. The type of linear equation used in this study is a system of linear equations with three variables. The participants of this study were three students of grade 10 of senior high school. This research was a qualitative case study. The instruments used were the test, questionnaires, and interview. The result obtained was participants of the high group tended to easily understand the problem, easily to remember similar problem, know what step used to solve the problem, measure the time needed to solve the problem, but not yet able to evaluate the answer correctly. The participant of the middle group was not likely to be difficult in understanding the problem, easy to remember similar problems, able to measure the time needed to solve the problem, but had not been able to determine the initial step and evaluate the answers correctly. While the subject of the low group tended to be incapable of understanding the problem, considering similar problems, knowing the initial steps used to do the problem, measuring the time needed to solve the problem, and not being able to evaluate the answers correctly. When solving linear equations problems, participants had different difficulties. The type of difficulties possessed by participants who had good metacognitive skills tended to be from memory factors, whereas participants who had not good metacognitive skills tended to derive from initial idea factors, correct step determination, and memory of concepts. Thus, the researcher suggests the need to conduct further research with more participants.

Keywords: linear equations, metacognitive skills, type of difficulties

1. INTRODUCTION

Mathematics is an important science that must be possessed by humans. Mathematics is used as one of the provisions of students after completing the school in order to survive in the community. Tok (2013) says that mathematics is needed by students when they are going to find a job. In addition, mathematics is a science which supports the other types of science. For example, physics, chemistry, electronics, and accounting will require mathematical skills in learning. So, understanding and mastery of mathematics are needed by an individual.

Cowan (2006) shares the mathematical knowledge learned by students into four contents. The contents are the skills, concepts, strategies, and techniques. These mathematical components are acquired by students in learning before applying them in the real life. Ozsoy (2011) states that learning will be effective if students are doing the learning process consciously. Such learning can only be done when the learning process contains metacognitive components. However, learning in general only focuses on getting the results of

calculations. As expressed by Tok (2013) that instruction in the class focuses only on the science of mathematics, but neglects the metacognitive component in solving the problem. Meanwhile, metacognitive instruction might ease the students to understand the meaning of the problem and determine strategies to solve problems in accordance with the ability they already have. One of the learning approaches that is used in Indonesia is scientific approach.

Hosnan (2013) argues that the material taught in the execution of a scientific approach is closely related to concrete facts or phenomena. Learning encourages students to think critically, creatively, analyze and modify, and apply learning materials, encourage students to think hypothetically, rationally and objectively, based on empirical concepts, and theories and facts that can be accounted for. The learning objectives are simple but meaningful and clear but still interesting. This will only be achieved if students follow the learning process consciously. Successful students are those who can determine when they need to be strategic and when they do not (Eggen and Kauchak 2001). So, the science delivered by the teachers is able to develop and improve the ability of students, especially on the ability of mathematics. A student must also be able to control his or her own cognitive process in order to be able to control and realize its shortcomings. Thus, students will easily choose the right strategy to solve a problem.

One of the mathematical abilities aiming to know and control the cognitive processes that have been done is the metacognitive ability. In the opinion of Flavell (1979), metacognition is a system useful in coordinating information, experiences, strategies, and objectives. Saengul & Katranci (2012) explain that metacognition, which means thinking about what is thought, generally includes various skills related to thinking and learning, which are reflective thinking, critical thinking, problem-solving, and making a decision. Biryukov (2004) explains that metacognition is a key factor in the problem-solving process. While Sengul & Katranci (2015) explain that metacognition includes strategy analysis that is used although the chosen strategy is not necessarily will lead to achieving the goal to be achieved. Jacobse & Harskamp (2012) argue that managing the problem-solving process while analyzing a task, making the plan and implementing it are also a part of metacognition. Based on the previous description, it is clear that essential metacognitive abilities are owned by every student.

Simons (2001) divides metacognitive ability into three components namely metacognitive knowledge, metacognitive skills, and metacognitive conceptions. Metacognitive knowledge is described as a deep knowledge and understanding of cognitive processes and products. Metacognitive skills are described as voluntary control people have over their own cognitive processes. While the metacognitive concept is a set of metacognitive component in which it classifies as a part of metacognitive knowledge. These three metacognitive components are closely related to other cognitive abilities in students. Metacognitive control skills consist of mental operations in metacognitive processes and can be defined as the ability to use metacognitive knowledge strategically to achieve cognitive goals (Desoete 2008; Schraw and Moshman 1995). According to Kapa (2001) cognitive task is affected by metacognition in each phase of the problem solving as it is given in Table 1.

Table 1. Metacognitive function classified according to the process phases

Solving-phase	The metacognitive function
Problem identification	Collecting data, coding, and remembering
Problem representation	Analogy, inference, imaginativeness, selective comparison, and combination
Planning how to solve	Integration, conceptualization, heuristic choosing, and formulating
Planning performance	Controlling and monitoring performance components of algorithmic mathematical knowledge and appropriate rules
Evaluation	Adjusting and contradicting a few possible or suggesting alternative solution methods

Based on the results of interviews with researchers at a high school towards the teachers of mathematics, most students do not have awareness in controlling their cognitive abilities. Some students even unable to solve a given problem, they often use their own way. The way that they used is not in accordance with the mathematical concepts that have been given. It is probable that the problem-solving ability of the students is not good enough. It is due to a reason that metacognitive and problem-solving abilities have a close relationship. If the students' metacognitive skills are still low, then the student will find it difficult to solve the problems it faces. Sengul & Katranci (2015) explain that students who use their skills such as metacognitive skills will be more successful in solving mathematical problems. Some researchers conducted research

focusing on metacognitive abilities.

There is a research conducted by Oszoy (2011) which shows a significant positive relationship between metacognitive ability and mathematical achievement. Not only in mathematical achievement, but metacognitive ability also can equally work. Verschaffel (1999) revised as students developed an understanding of the given problem and on the final outcome in interpreting and rechecking the calculations made, the metacognitive process involved in the problem-solver and metacognitive abilities as well. In solving a problem, metacognitive skills are needed in order to solve problems appropriately. Based on the description above, this research can analyze metacognitive skills in solving linear equation problems and identify type of difficulties during solving the problems.

2. METHOD

2.1. Design of the Research

This study is a qualitative study with case study approach. Creswell (2007) defines a case study approach as a research approach in which the researcher conducts an investigation of a bounded system or multiple bounded systems over time, through detailed, in-depth data collection and involves various sources of information and reporting the case description in context. This research was conducted in Semarang, Indonesia. The participants in this research were three students of grade 10 of senior high school in this area. The participants were selected purposively. Each student was drawn from a group of high, medium, and low. Grouping was based on the consideration of the math teacher teaching in the classroom.

2.2. Data Collection Tools, Collecting Data and Data Analysis

All participants were asked to solve two problems of linear equations prepared by the researchers. In the first problem, the solution can be found by using a system of linear equations such as elimination and substitution. In the second question, students must be able to find a comparison relationship with the system of linear equations. Once they found a connection, then the student will be able to solve the problem of number two. The second problem requires a little thought process than the first, because it requires more knowledge and has the dimension of information transfer. In line with the objectives of the study, as soon as the students completed both problems, students were asked to fill out the questionnaire. The questionnaire used was 'Using Metacognitive Skills Questionnaire', which was adapted from Biryukov (2004). The questionnaire consists of 14 questions that define cognitive and meta-cognitive behavior, which students do when solving mathematical problems. Students were given 30 minutes consisting of 20 minutes working on the problem and 10 minutes to answer the questionnaire. Data were obtained as a result and analyzed descriptively. In the first phase of analysis, the participants answered to solve linear equation problems and strategies in solving the problem. Then, the questionnaires item can be used as an interview guide. In addition, questions used as an interview guide coupled with questions relating to the difficulties experienced by students in solving the problem. After conducting in-depth interviews, the transcripts were analyzed and the results of the descriptions were clearly described.

3. RESULT AND DISCUSSION

All students in one class of 15 students had already answered two linear equation problems. After completing the two questions, students continued by answering the questionnaires prepared by the researcher. Then from 15 students were selected 3 students in which each of the high, medium, and low basic mathematics abilities. The selection of the subject was also considered the advice of the teacher who teaches the subject. Next, the answers from the three subjects about the system of linear equations of three variables and questionnaire answers were analyzed.

3.1 High Group Participant

Based on test results and questionnaires, high group subject has been able to solve number one of the problem and answer the questionnaire. Problem number one was done using the correct step but in the final calculation, S1 has not been able to evaluate the answer correctly (Figure 1). It is already stated that metacognitive skills are closely related to problem-solving abilities. As Verschaffel (1999) argues that when students build an understanding of the problems given and on the final results in interpreting and rechecking the calculations that have been done, the metacognitive process is involved in the problem solver. As S1 built understanding, wrote answers, and interpreted the number one, S1 has used his metacognitive skills. It was proven in the interview, S1 stated that he asked questions for himself when he did the answer. Questions like, how do I get started, using what concept to solve it, and what next steps should I do. Subject S1 wrote the answer "not sure" on the questionnaire. It stated that she understands what is asked on the

matter. Whereas the interview result stated that S1 has known and understood what is asked by the problem. S1 also admitted that he had checked the answer he had written. However, in reality S1 has not been able to write the right answer.

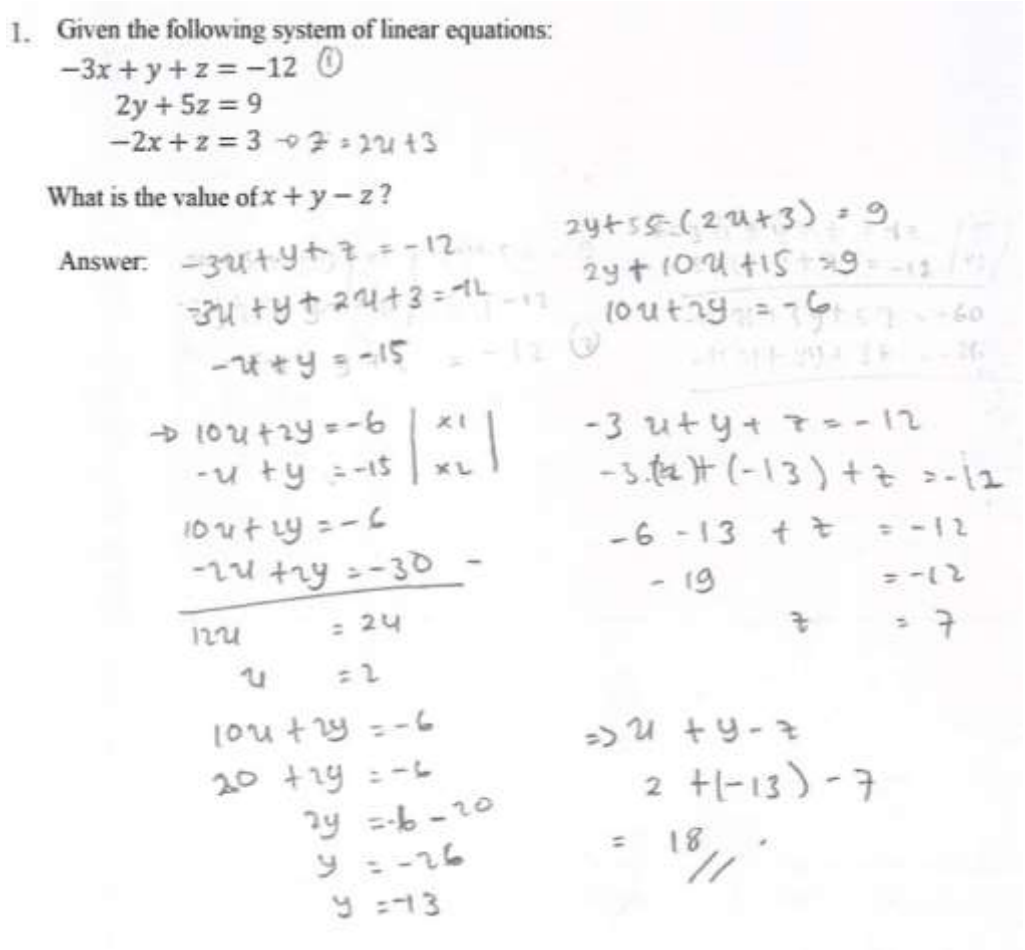

Figure 1. *Answer of S1 number 1*

This does not mean that the ability of metacognitive skills of S1 does not exist. As explained by Sengul & Katranci (2015), metacognition includes a strategy analysis that is used although the chosen strategy will not necessarily lead to achieve the objectives to be achieved. S1 has used the strategy and analyzed the steps that she would use to solve the problem. However, there were several factors that hinder it. Difficult factors for example, based on the results of interviews and observations from researchers, S1 had difficulty that came from difficult factors considering the concept used. S1 has mastered the concept, but when making a long-saved memory calls, S1 still felt difficult.

As for the question number two, S1 had tried but he had not managed to finish it. S1 tried to remember how to associate the concept of comparison with linear equations of three variables, but the process of redialing the stored memory has failed. Thus, S1 used the way that passes through his mind. The interview result stated that S1 had difficulty on this number two. The difficulty factor came from memory. S1 memory tended to be weak for the problem number two. During the interview, the researchers gave examples of comparative related questions, and S1 was able to answer them correctly. This indicates that S1 is actually capable, but still needs more effort.

3.2 Middle Group Participant

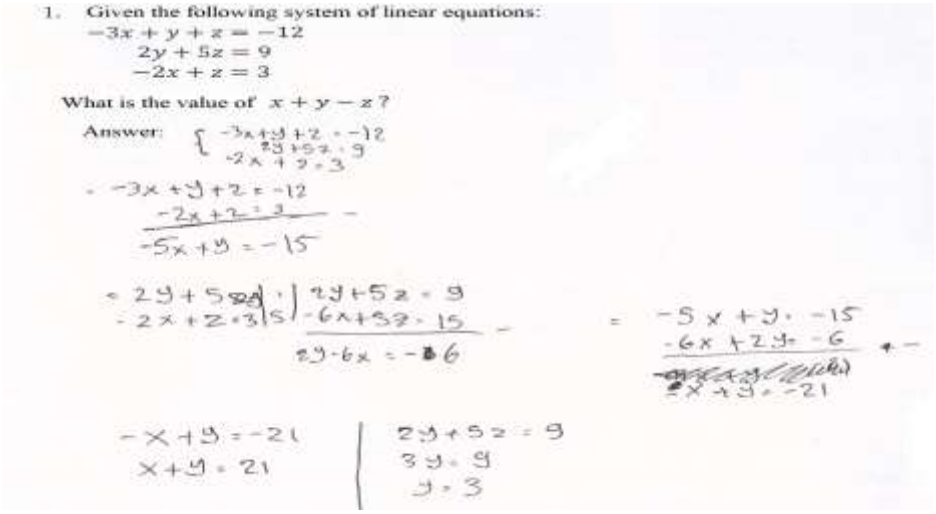

Figure 2: ***Answer of S2 number 1***

The subject of S2 has not been able to complete the two questions that have been given. S2 was only able to complete half of the question number 1 (Figure 2). From the results of questionnaires and interviews, S2 claimed that he had asked herself how to do it, but S1 did not continue the completion of number one. S2 admitted that he did not know what steps to use. This indicates the ability to master the concept of S2 is still lacking, so that impact also on other skills especially his metacognitive ability. S2 had not been able to evaluate its own answers. This is seen in the answers to questionnaires and S2 interviews. To solve the problem number one, it requires mastery of the concept of number operations. As seen on the answer sheet S2 on problem number one, S2 was still not capable of meticulous in operating numbers. Based on answers to questions, questionnaires, and interviews, and the analysis of the difficulties experienced by the S2 is on the factors namely the initial steps used to solve the problem, but S2 had not been able to continue to get the solution of the problem.

As for the number two, S2 has not been able to find the first step to solve the problem. S2 had not understood using what concept to answer the question number two. S2 could not make the plan and implement it to solve the problem. Jacobse & Harskamp (2012) argue that managing the problem-solving process while analyzing a task, making the plan and implementing it is also a part of metacognition. This indicates the lack of metacognitive skills owned by S2. When given a sample of comparative questions, S2 was confuse and had difficulties. Same as with S1, S2 also failed to recall the memory that has been stored. From the results of the analysis, S2 has difficulty recalling the concept used and has been able to find the first step that should be used but has not been able to solve it to get the right solution.

3.3 Low Group Participant

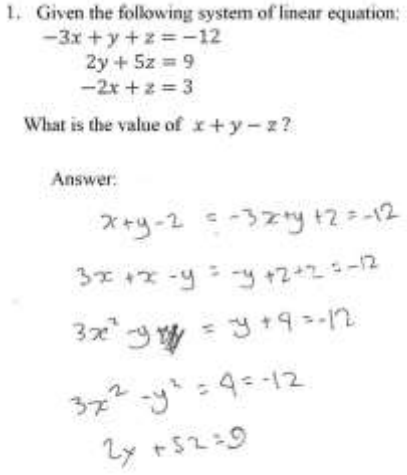

Figure 3. *Answer of S3 number 1*

Based on the answers to questions, questionnaires, and interviews, the S3 subject had not been able to solve both problems. S3 really cannot solve it. S3 has not been able to choose the right strategy. Seen from the answer of S3 (Figure 3), S3 had not mastered and understood what had been written. S3 also did not understand what the usefulness of the same symbol with. S3's prerequisite ability is also low, it is proved when asked: "Where did you get $3x + x = 3x^2$?" S3 replied that it is true, $3x + x = 3x^2$. Researchers tried to explain again about algebraic operations. The explanation that the researcher gave was more than four times in order to make S3 reconsider about algebraic operations. Then after S3 understood, the researcher gave back the form of the problem about the number operation, S3 still could not be thorough calculate the given problem.

Subject S3 filled the questionnaire with "not sure" as much as 5, "yes" as much as 6, and "no" as much as 1, and as many as 2 questions S3 did not answer it. Based on the explanation that has been presented, it indicates that the metacognitive skills of S3 are very less, which is also influenced by the problem-solving ability that S3 had. S3 said that he did not understand exactly about the problem given. After doing interview with S3, the researcher concluded that S3 had difficulties derived from inability to understand the problem exactly, prerequisite knowledge, understanding the concept of linear equations of three variables, and accuracy. While the difficulties experienced with regard to metacognitive skills is the difficulty in remembering the concepts used, the difficulty in choosing the first step that will be used in solving the problems given, and choose the right step to solve the problem.

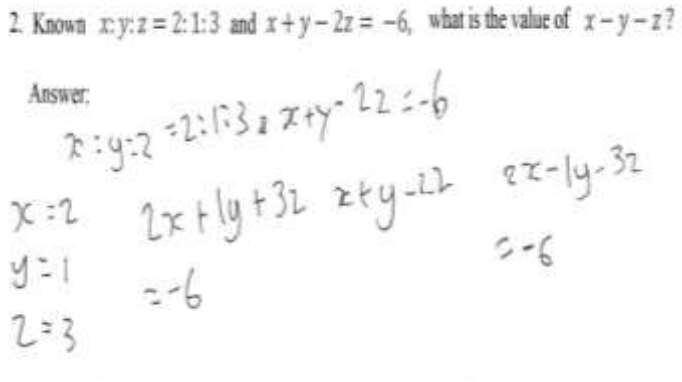

Figure 4. *Answer of S3 number 2*

As for the number two (Figure 4), S3 did not answer the given questionnaire. S3 admitted that he did not understand the problem. S3 did not find the right strategy to solve it. S3 was also not aware of any concept that was used to solve the problem number two. After the interview, it turned out that S3 used his own way to solve the problem number two. The way was not in accordance with the concept of mathematics. Basically, S3 thought that it is more difficult to think the correct concept, better to use the concept itself. Although S3 claimed to have asked himself how to do it, what concepts were used, and what steps were used to solve problems, it does not mean that he has high metacognitive skills. As Dawson (2008) argues, if individuals have good metacognitive skills, then the individuals will also be better at solving problems, making decisions, and thinking critically than others. This indicates that S3's metacognitive skills are still not good. S3 has not been able to make decisions correctly and have not thought critically to solve problems.

Metacognitive skills of S3 that has not been good was also lead to S3's difficulties to remember the concept to solve the problem, determine the initial strategy used, and to choose the right step to solve the problem. Based on the interview, S3 admitted that he was less interested in mathematics. This is because to answer a question, it needs some concepts and there are several ways to solve them. Indirectly, S3 revealed that he has difficulty coming from factors recalling the concept given, choosing the first step that will be used in solving the problems given, and choosing the right step to solve the problem.

4. CONCLUSION

The analysis of linear equation problems, questionnaires, and in-depth interviews, revealed about mathematical problem solving processes evidence of a metacognitive framework and demonstrate engagement of various metacognitive skills of the students. The results showed, that the metacognitive skills of the subject of the high group tended to be easy to understand the problem, easy to remember similar problems, knowing what initial steps used to do the questions, able to measure the time needed to solve the

problem, but not yet able to evaluate the answers correctly. The subject's metacognitive skills of the moderate group tended not to be difficult in understanding the problem, easy to remember similar problems, able to measure the time needed to solve the problem, but have not been able to evaluate the answers correctly. The subject's metacognitive skills of the low group tended to be incapable of understanding the problem, considering similar problems, knowing the initial steps used to do the questions, measuring the time needed to solve the problem, and not being able to evaluate the answers correctly. According to Kazemi, Fadee, and Bayat (2010) the comparison of student's metacognitive skills shows that students with higher levels of metacognitive ability perform better in problem solving tasks. Based on the results, the subject of the high group has the higher level of metacognitive ability then the low group. The answer of linear equation problems showed that the high group is more be able to solve problems than middle and low group. This indicates that metacognitive skills have a role in the problem solving process.

Sengul & Katranci (2015) explained that students who use their metacognitive skills will be more successful in solving mathematical problems. According to Kesici, Erdogan, and Ozteke (2011), they explain that metacognitive awareness strategy is significant predictor of mathematics achievement. It can show in performance of students in solving problems. Students who have the metacognitive awareness strategy can solve the problem easily. Indirectly, it can influence of mathematics achievement students. The study of Oszoy (2011) showed a significant positive relationship between metacognitive ability and mathematical achievement. Not only in mathematical achievement, but metacognitive ability also can equally work in both.

One of the important results of this study is about student's difficulties. The most prevalent of student's difficulties in linear equation problems solving are inability to recall the concepts used, to determine the first step to begin the solution, and to determine the next step after finding the initial step. Kazemi, et.al (2010) state that the most prevalent of student's difficulties in combinatorics problems solving as follow: inability to understand the problem exactly, inability to adapt the prior problems to task and subsequently choosing the wrong strategy for solving the problem, not having prior suitable information and inability to recall the formulae, Inability to be sure of the correctness of their find answers.

To summarize, the researchers have some suggestions for further research. In this study, subjects in this research were only three students. So, for further research it is expected to involve more subjects, so the results will be more diverse. Furthermore, the researcher in this study only focuses on metacognitive skills, perhaps for further research the field can be expanded.

ACKNOWLEDGMENT

This research is fully funded by Indonesia Endowment Fund for Education (LPDP) Scholarship which is given by Indonesian Government through the Ministry of Finance.

REFERENCE LIST

Biryukov, P. (2004). Metacognitive Aspects of Solving Combinatorics Problems. *International Journal in Educational Mathematics,* 74. March 29th.

Cowan, P. (2006). *Teaching mathematics: A handbook for primary and secondary school teachers*. USA: Routledge.

Creswell, J.W. (2007). *Qualitative Inquiry & Research Design: Choosing Among Five Approaches.* California: SAGE publication.

Dawson, T. L. (2008). *Metacognition and learning in adulthood. Prepared in response to tasking from ODNI/CHCO/IC Leadership Development Office*, Developmental Testing Service, LLC.

Desoete, A. (2008). Multi-method Assessment of Metacognitive Skills in Elementary School Children: How You Test is What You Get. *Metacognition and Learning*, 3(3), 189–206.

Flavell, J. H. (1979). Metacognitive and Cognitive Monitoring: A New Area of Cognitive Developmental Inquiry. *American Psychologyst*, 34, 906-911.

Eggen, P. & Kauchak, D. (2001). *Educational Psychology*. New Jersey, NJ: Merrill Prentice Hall.

Hosnan, M. (2013). *Pendekatan saintifik dan konstektual dalam pembelajaran abad 21*. Jakarta: Galia Indonesia.

Jacobse A. E., & Harskamp E. G. (2012). Towards Efficient Measurement of Metacognition in Mathematical Problem Solving. *Metacognition Learn*. 7(2):133–149.

Kapa, E. (2001). A Metacognitive Support During the Process of Problem Solving in A Computerized Environment. *Educational Studies in Mathematics*, 47(3), 317-336.

Kazemi, F., Fadee, M. R, Bayat, S. (2010). A Subtle View to Metacognitive Aspect of Mathematical Problem Solving. *Social and Behavioral Sciences*.8. 420-426.

Kesici,S., Erdogan, A., Ozteke, H. I. (2011). Are the Dimensions of Metacogntive Awareness Differing in Prediction of Mathematics and Geometry Achievement? *Social and Behavioral Sciences*. 15. 2658-2662.

Ozsoy. G. (2011). An Investigation of the Relationship Between Metacognition and Mathematics Achievement. *Asia Pacific Education*, 12, 227-235.

Simon, D. A., & Bjork, R. A. (2001). Metacognition in motor learning. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *27*(4), 907.

Saengul, S.& Katranci, Y. (2012). Metacognitive Aspects of Solving Function Problems. *Social and Behavioral Sciences*, 46, 2178-2182.

Saengul, S.& Katranci, Y. (2015). Meta-cognitive Aspects of Solving Indefinite Integral Problems. *Social and Behavioral Sciences*, 197, 622-629.

Tok, S. (2013). Effect of The Know-Want-Learn Strategy on Students, Mathematics Achievement, Anxiety, and Metacognitive Skills. *Metacognition Learning*, 8(2), 193-212.

Verschaffel, L. (1999). Realistic Mathematical Modelling and Problem Solving in The Upper Elementary School: Analysis and Improvement. *Teaching and learning thinking skills: context of learning*, 215-240.