MATHEMATICAL KNOWLEDGE AND HIGHER ORDER THINKING SKILLS FOR TEACHING ALGEBRAIC PROBLEM SOLVING

Nor'ain Mohd Tajudin*
*Assoc. Prof., Sultan Idris Education University, MALAYSIA, norain@fsmt.upsi.edu.my

Abstract

Solving algebraic problems is a complex process that is influenced by various factors including teachers' instruction. The present intense interest in research on teachers' knowledge and proficiencies demands that future problem-solving research pay close attention to the mathematical and pedagogical knowledge and proficiencies a teacher should possess. This study explores the Prospective Mathematics Teachers' mathematical knowledge for teaching algebraic problem solving and examines the extent to which the prospective teachers integrate the Higher Order Thinking Skills (HOTS) in teaching algebraic problem solving. The descriptive quantitative approach and the case study model were used in this study. The study was conducted with 66 Prospective Mathematics Teachers (PMT) who were undergoing their practical teaching in secondary schools, Malaysia. Out of 66 PMT, only three of them were chosen in order to get information related to what extent does PMT integrate HOTS in teaching algebraic problem solving. The data for this study was obtained using the written task-based questionnaire. The data obtained were analysed in accordance with the content analysis by focusing on the issues related to mathematical knowledge for teaching, heuristics or strategies during solving algebraic problem solving, and ways of PMT integrate HOTS in teaching algebraic problem solving as highlighted in the literature. Findings showed that a number of strategies were employed by PMT in teaching algebraic problem solving. There were seven categories that emerged as their dominant ways of thinking strategy in teaching how to solve the problem. However, two main strategies that were used by PMT are exploratory and Polya's methods. PMT in this study tended to explain everything in solving the algebraic problem. Only one PMT employed a group work method and another PMT employed a practical work method. In addition, two PMTs used questioning technique in teaching algebraic problem solving. These findings indicated that PMT in this study were not familiar with algebraic problem-solving methods (e.g., heuristics, strategies) that are accessible to students. Findings showed that the three PMT that were chosen in this study had established HOTS in teaching the algebraic problem solving. Their activities of teaching and learning the problem involved the link between Representation, Concept and HOTS. Our analyses shows that the relation between the above three key constructs are complex and that PMT need support in understanding these relationships in the context of non-routine problems.

Keywords: Mathematics Teachers, Mathematical Knowledge for Teaching Algebraic Problem Solving, Higher Order Thinking Skills, Representation, Mathematical Concepts.

1 INTRODUCTION

Research on mathematics education has provided valuable information about problem-solving instructional in guiding the design of worthwhile mathematical learning activities (Schoenfeld, 2013; Lester, 2013). Despite these decades of research and associated curriculum, teaching and learning development, it seems that students' problem solving abilities still require substantial improvement especially given the rapidly changing nature of today's world (Santos-Trigo & Moreno-Armella, 2013, Ministry of Education, 2013).

Findings of international studies such as Trends in Mathematics and Science Study (TIMSS) (Mullis, Martin, Foy & Arora, 2012) and Programme of International Student Assessment (PISA) (Organisation for Economic Co-operation and Development (OECD), 2013) showed that most high school students in Malaysia continue to perform at less than satisfactory levels. They have not reached a satisfactory level of higher order thinking, specifically in mathematical problems solving. Towards that end, Malaysian National Education Transformation Plan 2013-2025 (Ministry of Education, 2013) has emphasised the need to develop quality mathematics teachers for the future who are capable of making innovative and creative pedagogical decisions in varying contexts.

For example, in the analyses of Malaysian students' performance in TIMSS in a number of years, Mullis et al. (2012) found that only 2-10% of the students are capable of interpreting the information, drawing conclusions and generalization in solving complex problems – activities that collectively reflect low levels of activation of cognitive processes in mathematical problem solving. Mullis and colleagues also showed that 60% of students achieved the low international benchmark. These results suggest that the students understand the basic mathematical concepts but, in general, they are not able to transfer that knowledge to non-routine problem situations (Ministry of Education, 2013). Likewise, another international study, namely, the PISA 2009 showed that Malaysia students' mathematical performances were located in the bottom one third of all the 74 participating countries (Walker, 2011).

As in the TIMSS study, PISA's report for mathematics achievement showed that only a small proportion (8%) of Malaysian students achieved advanced levels of thinking. Overall, trends in TIMSS and PISA provide evidence of Malaysian students' continuing difficulty in solving mathematical tasks which involve complex interpretation and synthesis – key aspects of mathematical problem solving. The solution of complex mathematical problems involves the transfer of prior learning to new contexts, and this transfer, we argue, can be facilitated by the acquisition of appropriate mathematical problem solving instruction.

2. CONCEPTS UNDERLYING THE STUDY

2.1 Mathematical Knowledge for Teaching

Mathematical knowledge for teaching (MKT) refers to a specific body of knowledge that mathematics teacher must poses and uses in the classroom to produce instruction and students' growth (Hill, Ball & Schilling, 2008). Several studies suggest that the nature, depth, and organization of teacher knowledge influences teachers' presentation of ideas, flexibility in responding to students' questions, and capacity for helping students connect mathematical ideas (e.g., Ball, 1988). Mathematics teachers are a special class of users of mathematics; the knowledge they need to teach mathematics goes beyond what is needed by other well-educated adults, including mathematicians (Ball, Thames, & Phelps, 2008). Further, Hill and colleagues associate higher levels of MKT to greater student learning gains, underscoring the importance of teachers' MKT and its impact on what students learn in the classroom (Hill et al., 2008).

Consistent with Shulman (1986), Ball, Bass, and Hill (2005) endeavoured to answer the same question about teacher knowledge: What do teachers do in teaching mathematics, and in what ways does do they demand mathematical reasoning, insight, understanding, and skill? Ball, Bass, and Hill expanded and re-partitioned Shulman's teacher content-knowledge divisions and named it as MKT (Ball, Thames, & Phelps, 2008).

Theoretically, the MKT construct follows Shulman's (1986) efforts to define the theories concerning subject matter knowledge (SMK) and pedagogical content knowledge (PCK). A proposed strand of the MKT composed of each of the six portions of the oval. The left side of the oval, labelled SMK contains three strands: Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Knowledge at the Mathematical Horizon. Whilst the right side of the oval, labelled PCK composed of another three strands: Knowledge of Content and Students (KCS) and Knowledge of Teaching and Content (KTC) and Knowledge of Curriculum. See Figure 1 for a comparison of the frameworks from Ball, Bass, and Hill (2005) and Shulman (1986).

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The MKT is the key idea of knowledge that mathematics teachers should have in teaching. However, in teaching mathematical problem solving or specifically in this study, the algebraic problem solving and integrating HOTS requires another methodology that can be utilised by teachers better understand the role of HOTS in empowering students move from lower levels to higher levels of cognitive funtioning in the context of complex tasks. Thus, next section will highlight this concept of methodology.

2.2 Relationship between Higher Order Thinking Skills (HOTS) and Task Variables

One major instructional goal of mathematics education is fostering students' ability to think at higher levels (National Council of Teachers of Mathematics, 2000; Gonzalez, 2012). However, teaching for the development of HOTS has been argued to be difficult and challenging (Ravitch, 2010) because the nature of HOTS is not clear. Therefore, describing HOTS is crucial in order for teachers to weave these skills into the learning experience of the students.

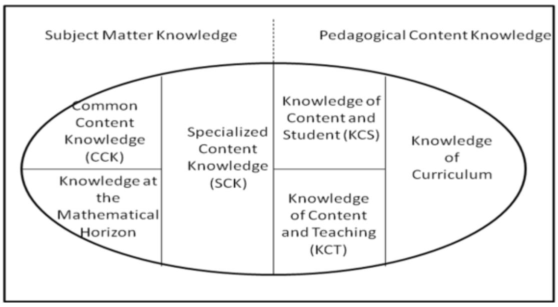

Figure 2.1: Domain map of mathematical knowledge for teaching
(Source: Adapted from Ball, D. L., Thames, M. H. & Phelps, G. (2008) Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), p. 403.

There are many ways of conceptualizing the range of skills underlying HOTS as played out in mathematical performance. Resnick (1987) put forward the concept of HOTS as non-algorithmic, complex, yields multiple solutions, requires the application of multiple criteria, self-regulation, and often involves uncertainty; which suggest some characteristics of HOTS skills. In the same vein, Stein and Lane (1996) emphasise that HOTS involved "the use of complex, non-algorithmic thinking to solve a task in which there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instruction, or a worked out example" (p. 58). Similarly, the National Council of Teachers of Mathematics (1989) describes HOTS supporting the solution of a non-routine problem. A non-routine problem is defined as "a problem involving - a situation in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed" (National Council of Teachers of Mathematics, 1989, p. 10).

In Malaysian contexts, the Curriculum Development Centre (2013) defines HOTS as the potential to apply knowledge, skills and values for reasoning and reflection for solving problems, making decision and capable and innovative in creating things. The description of HOTS is similar with that of Thinking Schools International (Burden, 2010) which states HOTS involves fundamental cognitive processes for generating and organizing information, skills of analysis and synthesis, and processes of creativity and evaluation. In general, there's a common understanding that students are giving limited exposure to HOTS and that HOTS involved complex process such that analyzing, generating, evaluating and making reflection.

One major concern of this study is to elucidate the role of HOTS in helping students tackle cognitively demanding problems in mathematical problems. Two key dimensions of these context problems were identified. Firstly, the *Content* that is embedded in the problems was analysed. Secondly, the construction of potentially useful representations of the problems was identified. The links between the two above dimensions of the task and the HOTS were examined. Figure 2 provides a schematic representation of these key constructs. As indicated in the figure, all three constructs are informed and informed by each other.

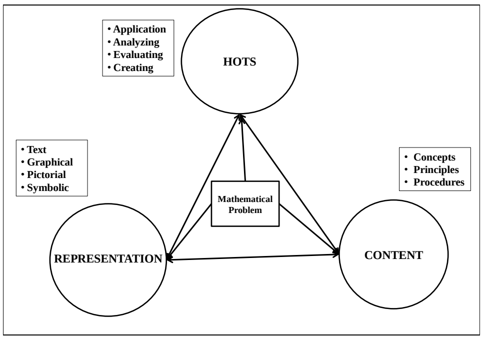

Figure 2: Relationship between HOTS and Task Variables

3 OBJECTIVES OF THE STUDY

The aim of the study is to explore the prospective mathematics teachers' mathematical knowledge for teaching algebraic problem solving. In this study, one particular type of mathematical knowledge for teaching will be considered namely the heuristic or strategy for teaching algebraic problem solving. In addition, this study will also examine the extent to which the prospective teachers integrate HOTS in teaching algebraic problem solving. Specifically, this study sought data relevant to the following three research questions:

1. What are the heuristic or strategies suggest by the prospective mathematics teachers in teaching algebraic problem solving?
2. To what extent does prospective mathematics integrate higher order thinking skills in teaching algebraic problem solving?

4 METHODOLOGY

The case study model, which is among the qualitative, naturalistic research perspective (Creswell, 2008) was used in this study which focusing on capturing and interpreting participants' thinking about a phenomenon, namely, the mathematical knowledge for teaching. The qualitative research method of case study proved to be an appropriate research design for exploring aspects of teaching and learning (Yin, 1994).

4.1 Participants

The study was conducted with 66 prospective mathematics teachers (PMT) who were undergoing their practical teaching in secondary schools, Malaysia, in the seventh semester of their 4-year degree Mathematics Education Programme. These PMT had taken all mathematics education courses except the Reflection Seminar Course and had completed all of their mathematics required for the programme. The Reflection Seminar Course will be taken in their final eighth semester because this course provides the opportunity to the students to evaluate and discuss their experience related to practicum. Thus, they had instruction or theory on mathematical problem solving as well as their content mathematics knowledge. Out of 44 PMT, only three of them were chosen in order to get information for research question number three which related to what extent does PMT integrate higher order thinking skills in teaching algebraic problem solving. This was due to reason such as by examining the heuristics or strategies codes developed from 44 data of written task-based questionnaire. As the data were returning to no new codes, and therefore were saturated (Strauss & Corbin, 1998). The conclusion was that for this study with a high level of homogeneity among the 44 PMT, a sample of three written task-based questionnaire data may [be] sufficient to enable development of meaningful themes and useful interpretations.

4.2 Instrument

The data for this study was obtained using the written task-based questionnaire [WTQ]. All PMT were required to respond on one question to reveal PMT's heuristics or strategies of teaching algebraic problem solving. The question is shown in Figure 3 This question was selected in order to ensure that *HOTS*, *content* and *representation* constructs are activated by PMT as they could explain their heuristics or strategies by considering all three key constructs interwoven and interdependent on each other. WTQ was validated by obtaining the opinions of two mathematics educators who were experts in the field. This is because the

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content validity of measurement tools is subjective and based on expert opinion. The content validity of the questionnaire was established in accordance with the conducted studies and expert opinions.

> Given ABCD is a piece of rectangular paper with an area of 28 cm^2, a width of 7*x* cm and a length of *y* cm. AEB is a semicircle-shaped cut from the paper. The remaining perimeter of the paper is 26 cm. Find the integer values of *x* and *y* (Use $\pi = \frac{22}{7}$).

Figure 3: Written Task-Based Question

4.3 Analysis of the Data

As elaborated above, the data for this study was obtained using WTQ. Once WTQ were returned to the researcher, they were coded as PMT1 until PMT44 on the above right hand corner of the questionnaire. The data obtained were analysed in accordance with the content analysis by focusing on the issues related to mathematical knowledge for teaching, heuristics or strategies during solving algebraic problem solving, and ways of PMT integrate *HOTS* in teaching algebraic problem solving as highlighted in the literature. The analysis began with open-ended coding (Strauss & Corbin, 1998) of the data. Categories and subcategories were formed in the analysis of the written task-based questionnaire by benefiting from the data of the pilot study and the literature. Furthermore, the categories and subcategories that had been formed by the researchers were examined by two experts and they were finalized.

The answers of each PMT were examined accordingly. As mentioned above, the answers of the PMT were analysed in the scope of the content analysis' model, which is considered to be a qualitative research method. This is because the content analysis makes it possible to give meaning to the obtained raw data; to arrange this data after the emerged case has become clear; and to reveal and concretize categories and codes.

With respect to research questions one - What are the heuristic or strategies suggest by the prospective mathematics teachers in teaching algebraic problem solving?, the coded information was categorised based on common themes and frequency of occurrence. The answers given by PMT in WTQ have been analysed according to the following categories of heuristics or strategies as suggested several scolars, for example, Polya, (1945) and Schoenfeld (1985), which also documented in Malaysian Mathematics Curriculum Syllabus (Ministry of Education, 2012). The strategies that could be used including trying a simple case, trial and improvement, drawing diagrams, identifying patterns, making a table, chart or systematic list, simulation, using analogies, working backwards, logical reasoning, and using algebra.

With respect to research question two - To what extent does prospective mathematics integrate higher order thinking skills in teaching algebraic problem solving?, the data were analyzed according to the description of analysis of tasks involving three concepts: Representation, Concepts and HOTs.

5 Results

The results are presented based on two research questions, namely (i) heuristics or strategies used by the PMT in teaching algebraic problem, and (ii) PMT integration of higher order thinking skills [HOTS] in teaching algebraic problem solving.

The study was conducted with 66 prospective mathematics teachers [PMT] who were undergoing their practical teaching in secondary schools, Malaysia, in the seventh semester of their 4-year degree Mathematics Education Programme. However, only 44 (66.67%) students were participated in this study. Majority of the participants (79.55%) were females in their early twenties. With regards to respondents in analyzing data for research question three, PMT5, PMT6 and PMT32 were purposely selected based on their various heuristics and strategies used in teaching the algebraic problem solving. All three were females with two Malays and one Chinese.

5.1 Strategies in Teaching Algebraic Problem Solving

There was variety in the nature of PMT strategy in teaching algebraic problem solving. There were seven categories that emerged as their dominant ways of thinking strategy in teaching how to solve the problem. Table 1 presented those categories with correspond frequency occurrences. Some of the PMT used more than one strategy in teaching the problem. The findings indicated that about three quarter (72.7%) of the PMT normally used the exploratory method in teaching the algebraic problem solving. Secondly, one quarter

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of them (25%) are comfortable with four step-by-step Polya's method. This indicated that most PMT were not familiar with algebraic problem-solving methods (e.g., heuristics, strategies) that are accessible to students. About 13% to 18% of the PMT using concrete materials such as the manila card and A4 paper, paper cutting strategy and drawing a diagram in explaining the question asked and understanding how to create two simultaneous equations to solve for *x* and *y*. Five percent of them integrated the questioning technique in order to elicit students' ideas, understanding of the question and to prompt them. Only one PMT tried to engage students using practical method and group work in teaching the algebraic problem solving.

In this article, two main strategies that have been employed by PMT will be explained; the Polya's and exposition methods. With regards to Polya's method, PMT5 will be the example, while PMT43 as an example of using exposition method.

PMT 5 exhibited the strategy of four step-by step method by firstly asking students to extract the information given in the diagram. Then, PMT 5 guided the students to plan a strategy such as finding two equations using the information given involving variables *x* and *y* and finally solve the simultaneous equation. Next, PMT 5 guided students to perform those strategies. Finally, PMT 5 made sure that the students checked their answers by substituting both sets of answers into one of the equations such that the correct solution must satisfy the equation. The same strategy was also employed by PMT 3, 4, 9, 10, 13, 18, 25, 35, 39, and 40.

Table 1: Category of PMTs' strategy in teaching algebraic problem solving

Category	PMT	Frequency (Percentage)
Four step-by-step Polya method	3, 4, 5, 9,10, 13, 18, 25, 35, 39, 40,	11 (25%)
Exposition	2, 6, 7, 8, 11, 12, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 41, 42, 43, 44	32(72.7%)
Paper cutting	1, 3, 7, 26, 31, 32, 38, 41	8 (18.2%)
Using concrete material	1,7, 8, 31, 32, 33, 41	7 (15.9%)
Questioning technique	8, 9	2 (4.5%)
Drawing a diagram	6, 13, 15, 27, 28, 29	6 (13.6%)
Practical method Group work	1 41	1(2.4%) 1(2.4%)

Other examples are PMT 2, 6, 7, 8 and etc. who exhibited the exposition method where they just demonstrated to students on how to work out the problem and students follow their instructions. For example, PMT 43 explained the first step to teach the algebraic problem solving is that teacher needs to show students to extract the main points given in the question. Then, PMT43 suggested that students could create formulas based on area of rectangular shape and perimeter of the leftover cutting paper. The following step was that PMT43 showed to students how to solve the simultaneous equation.

A few PMT employed a combination strategy of exposition method, practical method, using concrete material, paper cutting to teach the algebraic problem solving. For example, PMT1 described that she will teach by drawing a diagram and paste it on the whiteboard. PMT3 explained that she will cut a semicircle that she drew on A4 paper to show the leftover diagram. On the other hand, PMT6 re-drew the diagram in the question on whiteboard and shaded the area with different marker pens. Another example is PMT7 who also re-drew the diagram on a manila card and will use this diagram to explain how to transform visual representation to symbolic forms.

5.2 Integrating HOTS in Teaching Algebraic Problem Solving

With respect to research question two - To what extent does prospective mathematics integrate higher order thinking skills in teaching algebraic problem solving?, I illustrate this by only considering data from three PMT. The PMT are PMT5, PMT6 and PMT32.

PMT5 is a Chinese female. In order to solve this problem, PMT5 described that she will requires students to identify the necessary elements and determine the links between these elements, a process that requires *Analyzing* – one of *HOTS*.

> "First of all, I will ask students to extract whatever information is given in the diagram...and the question is to find x and y...how do we form the equation? That is by analysing the information given."

Then, PMT5 guided students to translate the texts and diagram to symbolic representation (equation). This step of constructing involves *Creating*- another *HOTS*.

> "...and ask students to form the first equation...Secondly, I asked students to form the 2nd equation."

Through the creation of the symbolic representation, students could focus on the mathematical content that underpins the problem including rectangular shape, semicircle, perimeter, area and integer.

> "By looking at the first information area of rectangle = 28cm^2...Then students wills see that parameter of the leftover diagram is..."

On completing the solution, PMT5 asked students to engage in activity that examine the reasonableness of the values of *x* and *y* – *Evaluating (HOTS).*

> "However, if we substitute both set of answers into 7x and y which is the length and breadth of rectangle, we know that $7x > y$ where (7x is longer than y according to the diagram)."

The next step, PMT5 explained how the students could involve the use of a strategy where the students solve the equation by working from the known to unknown. The strategy involved restructuring the equation, substitution one equation into another equation, expansion and factorization. These forms of activities involve – *Application*, *Analyzing*, *Creating* and *Evaluating* – activities in *HOTS*.

> "...I explained that we have to restructure the equation...Now we have to substitute equation(3) into (1)...students have to expand the equation...I will repeat again factorization using cross method."

Overall, PMT5 exhibited the integration of HOTS throughout her teaching and learning processes of algebraic problem solving.

PMT6 is a Malay female and had Physics as her minor subject. In order to solve this problem, PMT5 described that she will use a strategy of drawing a diagram by re-drawing the diagram on whiteboard and labelling with different colour of marker pens. Re-drawing the diagram involves *Analyzing* information in the problem – one of *HOTS*.

> "I will use the diagrams. I will redraw the diagram in the question on the white board and label them using different colored marker."

Then, PMT5 guided students to translate the visual diagram to symbolic two symbolic equations. This step of constructing involves *Creating*- another *HOTS*.

> "I will ask students to define the length and width of the rectangle ... I will ask the students to rewrite the perimeter of the left diagram."

The next step, PMT6 explained how the students could solve the simultaneous equation by using substitution method. This step involves PMT5 engaged her students with activities such as restructuring the equation, substitution one equation into another equation, expansion and factorization. These activities involve – *Application*, *Analyzing*, *Creating* and *Evaluating* – which highlight *HOTS*.

PMT32 is also a Malay female and had Physics as her minor subject. In teaching to solve this problem, firstly, PMT32 described that she will ask students to create a rectangular shape ABCD on a piece of paper. Based on a key word "area of a rectangular", PMT32 asked students to construct a symbolic equation. This involves *Creating*- one of *HOTS*.

> "... I will ask students to build an ABCD square on a piece of paper ... then I will use the keyword or formula involved that is the formula of the area of the square ..."

Using the same strategy as above, PMT32 required students to cut the rectangular shape as in the problem. Based on a key word" perimeter, PMT32 told the students to find the second equation. This too involves *Creating* (*HOTS*).

> "I will ask the students to cut the semicircle ... I will use the information of the perimeter ... where I will form an equation again."

Finally, PMT32 explained that the students can find values of x and y by using the simultaneous equation method. Nevertheless, she did not explain in detail how to solve it. By assuming that students have prior knowledge of method in solving simultaneous equation, student may engage in activities such as restructuring the equation, substitution one equation into another equation, expansion and factorization. As mentioned above, these activities involve – *Application*, *Analyzing*, *Creating* and *Evaluating* – which highlight *HOTS*.

Overall, the three PMT had established *HOTS* in teaching the algebraic problem solving. Activities of teaching and learning the problem involved the link between *Representation*, *Concept* and *HOTS*.

6 Discussion

A number of strategies were employed by PMT in teaching algebraic problem solving. There were seven categories that emerged as their dominant ways of thinking strategy in teaching how to solve the problem. However, two main strategies that were used by PMT are exploratory and Polya's methods. PMT in this study tended to explain everything in solving the algebraic problem. Only one PMT employed a group work method (PMT41) and another PMT employed a practical work method (PMT1). In addition, two PMTs used questioning technique in teaching algebraic problem solving (PMT8 and PMT9). These findings indicated that PMT in this study were not familiar with algebraic problem-solving methods (e.g., heuristics, strategies) that are accessible to students.

Overall, the three PMT had established *HOTS* in teaching the algebraic problem solving. Activities of teaching and learning the problem involved the link between *Representation*, *Concept* and *HOTS*. The PMT required students to identify the necessary elements and determine the links between these elements, a process that requires *Analyzing* – one of *HOTS*. They guided students to translate the texts and diagram to symbolic representation (equation). This step of constructing involves *Creating*- another *HOTS*.

Based on the findings of this study, we conclude that teachers' professional programme in Malaysia must provide explicit instruction in MKT algberaic problem solving. In additon, the SCK should be emphasized explicitly as PMTs must know and understand it to enable them to teach effectively. Regular mathematics education courses should allocate time to support PMTs to reason without any constraints to produce correct or incorrect answers to predetermined outcomes. The current curriculum in promoting MKT is not grounded in a complete undesrtanding of what these knowledge are and how they are played out in algebraic prolem solving. Further research should be conducted to generate higher level of clarity about the roles of MKT in algebraic problem solving in mathematics learning.

Hill and Charalambous's research (2012) supports that Mathematical Knowledge for Teaching (MKT) contributed to instructional quality, it therefore would not seem unreasonable to suggest that if we want to improve teacher effectiveness the development of MKT is an important factor. At the very least, familiarity with this construct would allow teachers to reflect on the various domains that require development to foster PCK, and allow them the opportunity to strengthen any areas in which they may feel they are deficient. Classroom instruction can offer many opportunities to involve students in early grades in algebraic thinking. Since, generalizing is the heart of algebraic thinking (Carraher, et al., 2006), students should be encouraged to identify, extend, and generalize pattems (Smith, 2003). This type of instruction will help them develop "the 'habits of mind' people use when they think algebraically" (Johanning, 2004, P. 372). As suggested by Kaput and Blanton (2001) teachers should spot any opportunity for generalization, and encourage students to build generalizations of number properties and relationships.

All PMT should be familiar with algebraic problem-solving methods (e.g., heuristics, strategies) that are accessible to students. This involves mastering each problem solving strategies that were suggested by the Curriculum Development Centre, Ministry of Education (2012). The skill of problem solving could and should be taught – it is not something that you are born (Polya, 1945). Thus, Polya's Method should also be mastered by PMT as it formed the basis for any serious attempt at problem solving: understand the problem, devise a plan, carry out the plan, and look back (reflect).

We have demonstrated that HOTS are crucial for students to move from one phase to another phase by activating relevant Content and marshalling this information during the construction of rich and powerful Representation. Thus, teachers play a critical role in foregrounding and integrating HOTS, Content and Representation in the above problem contexts. Our analyses in this report shows that the relation between the above three key constructs are complex and that PMT need support in understanding these relationships in the context of non-routine problems. The conceptual framework provided in Figure 2 explicates the links between HOTS, Content and Representation and provides a useful guide for classroom practice. In order for teachers to orcharstrate these three co-dimensions, they need to undertake the critical analyses of the mathematical tasks in question.

Mathematics teachers also need guidance in lesson planning that articulate a clear understanding of what constitute HOTS instruction, and how to implement them in regular classroom. The results of our tasks analyses and HOTS provides an important starting point for designing future professional development programs for Malaysian mathematics teachers in adapting HOTS into secondary mathematics instruction.

7 CONCLUSIONS

This study indicated that PMT do seem to hold the "Mathematics Knowledge for Teaching". However, majority of the prospective teachers were not familiar with the best algebraic problem-solving methods that are accessible to students to teach the task. They were comfortable of using the exposition method by explaining how to work out on the task. In addition, they also irregularly used the four step-by-step Polya Method in teaching algebraic problem solving. In this study, a critical analysis of selected mathematical tasks and demonstration on how to better support students in the use of HOTS in making progress with such tasks has been presented. A methodology that can be utilised by teachers to better understand the role of HOTS in empowering students move from lower to higher levels of cognitive funtioning in the context of TIMSS and similarly demanding tasks has been shown. The methodology provides an important starting point for the design of future professional development programs for Malaysian mathematics teachers in articulating HOTS and implementing them in regular classrooms.

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REFERENCES

Ball, D. L. (1988). *Knowledge and reasoning in mathematics pedagogy: Examining what prospective teachers bring to teacher education*. Unpublished doctoral dissertation, Michigan State University, East Lansing.

Ball, D. L., Hill, H. H., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, Fall*, 14-46.

Ball, D. L., Thames, M. H. & Phelps, G. (2008) Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5):389–407.

Burden, R. L. (2010). What is a thinking school? *Creative Teaching & Learning, 1(2),* 22-26.

Carraher, D. W., Schlieamann, A. D., Brizuela, B. M., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education, *J. Res. Math. Educ.* 37, 87–115.

Creswell, J. W. (2008). Research design: Qualitative, quantitative, and mixed methods approaches, Oaks, CA: SAGE Publications, Incorporated.

Curriculum Development Centre (2012). *Kurikulum Bersepadu Sekolah Menengah, Spesifikasi Kurikulum Matematik Tingkatan 4*, Putrajaya: Bahagian Pembangunan Kurikulum, Kementerian Pelajaran Malaysia. Available online: http://www.gov.moe.my

Curriculum Development Centre (2013). *Kemahiran Berfikir Aras Tinggi dalam Sains dan Matematik (Higher Order Thinking in Sciences and Mathematics (HOTsSM).* Available online: http://www.gov.moe.my

Hill, H. C., Ball, D. L., & Shilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for Research in Mathematics Education, 39(4):372-400.

Hill, H. C. & Charalambous, C. Y. (2012) Teacher Knowledge, Curriculum Materials, and Quality of Instruction: Lessons Learned and Open Issues. Journal of Curriculum Studies, 44, (4), 559-576. http://dx.doi.org/10.1080/00220272.2012.716978

Johanning, D. I. (2004). Supporting the development of algebraic thinking in middle school: A closer look at students' informal strategies. *Journal of Mathematical Behavior, 23*(4), 371–388.

ISBN: 978-605-64453-3-0

Kaput, J. & Blanton, M. (2001)"Student Achievement in Algebraic Thinking: A Comparison of Third Graders' Performance on a State Fourth-Grade Assessment." In *Proceedings of the XXIII Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, edited by R. Speiser and C. Maher. Columbus, Ohio: ERIC Clearinghouse.

Lester, F. K. (2013). Thoughts about research on mathematical problem- solving instruction. In L. Santos-Trigo & L. Moreno-Armella, International Perspectives on Problem Solving Research in Mathematics Education, a special issue. The Mathematics Enthusiast, 10(1&2): 245–278.

Ministry of Education. (2013). Malaysia Education Blueprint 2013-2025 (Preschoool to Post-Secondary Education, Putrajaya, Malaysia: Kementerian Pendidikan Malaysia.

Mullis, I. V. S., Martin, M. O., Foy, P. & Arora, A. (2012). TIMSS 2011 International results in mathematics. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College. Reterieved 2 January 2013, from http://timssandpirls.bc.edu/timss2011/downloads/T11_IR_Mathematics_FullBook.pdf.

National Council of Teachers of Mathematics, (1989). *Curriculum and evaluation standards for school mathematics.*Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics, (2000). *Principles and standards for school Mathematics.* Edited by NCTM, Reston, VA: National Council of Teachers of Mathematics

Organisation for Economic Co-operation and Development (OECD). (2013). PISA 2012 Results. Retrieved 4 January 2013, from http://www.oecd.org/pisa/keyfindings/pisa-2012-results.htm.

Pólya, G. (1945). *How to solve it: A new aspect of mathematical method,* Princeton, USA, Princeton University Press.

Ravitch, D. (2010). *The death and life of the great American school system: How testing and choice are undermining education*. Philadelphia, PA: Basic Books

Resnick, L. (1987). *Education and learning to think*. Washington, DC: National Academy Press.

Santos-Trigo, L. & Moreno-Armella, L. (2013). International perspectives on problem solving research in mathematics education, a special issue. The Mathematics Enthusiast, 10(1&2): 3–8.

Schoenfeld, A. H. (1985). Mathematical Problem Solving. New York: Academic Press.

Schoenfeld, A. H. (2013). Reflections on problem solving theory and practice. In L. Santos-Trigo & L. Moreno-Armella, International Perspectives on Problem Solving Research in Mathematics Education, a special issue. The Mathematics Enthusiast, 10(1&2):9–34.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Smith, E. (2003). Statis and change: Integrating patterns, functions and algebra throughout the K-12 curriculum. In J. Kilpatrick, W. G. Martin, & D. Schifter(Eds.), A research companion to 'Principles and standards for school mathematics' (pp. 136–150). Reston, VA: National Council of Teachers of Mathematics.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50-80.

Strauss, A. L., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory.* , Thousand Oaks, CA: Sage Publications.

Walker, M. (2011). *PISA 2009 Plus results performance of 15-years olds in reading, mathematics and science for 10 additional participants*. Melbourne: Australia Council for Educational Research (ACER). Retrieved 30 August 2012, from http://research.acer.edu.au/cgi/viewcontent.cgi?article=1000&context=pisa.

Yin, R. K. (1994).*Case Study Research, Design and Methods,* 2nd ed. Newbury Park: Sage Publications.

ISBN: 978-605-64453-3-0