WORD PROBLEMS OF CONIC SECTIONS: ANALYSIS OF LINGUISTIC DIFFICULTIES

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Abstract

Language has become an essential factor in students' performance in solving word problems, especially in the topic that requires multiple representations of terms such as conic sections. This study describes findings on students' language difficulties in word problems with the topic of conic sections. A test in the form of word problems was constructed, validated to mathematics expert, and then given to 28 students of 11-grade. Student's work was analyzed from the perspective of linguistic difficulties by classifying each obtained case into three features of mathematics language difficulties, i.e. multiple semiotic systems, vocabulary, and grammar and syntax. It was found that multiple semiotic systems was the most found feature of linguistic difficulties, particularly in the difficulties of transforming language into mathematical symbol. Difficulty found in vocabulary feature was the difficulty in understanding technical term in conic sections, while in grammar and syntax feature was the difficulty in considering implicit meaning within word problem. This study concludes that the difficulties due to linguistics feature of mathematics are real and faced by students during word problem solving of conic sections. Those difficulties are found in various cases and the features correspond to them (i.e. multiple semiotic systems, vocabulary, and grammar and syntax) can be seen as separated or related to each other at the same time.

Keywords: language, mathematics word problems, conic sections, difficulty.

1 INTRODUCTION

Conic sections become the new topic included in mathematics content of the new 2013 Indonesian curriculum (Ministry of Education and Culture, 2013). The topic covers the analysis and description of parabola, ellipse, and hyperbola concepts. Beside the understanding of those concepts, the skills to apply the concepts into real context is also mandated by the curriculum as written in Indonesia content standard (Standar Isi) no. 21 as part of the new 2013 curriculum (Ministry of Education and Culture, 2016). It is stated that students should be able to identify whole or partial information from daily life problems (p.112) and use strategies to solve daily life problems (p.114). Thus, solving word problems in the topic of conic sections becomes an important issue to discuss.

Word problem has been considered difficult compared to other type of problem. It requires complex steps to solve (i.e. reading, comprehending, transforming into mathematical expression, processing the mathematics, interpreting result to context given, and evaluating the result) (Reys, Lindquist, Lambdin, & Smith, 2008; Ryan & Williams, 2007; Verschaffel et al., 2010). Thus, solving word problem will not only need mathematics skills, but also ability to make meaning from language, that is language skill. Some studies identified language difficulties in solving word problems, e.g. difficulties in translating the problem and how it is strongly correlated with the accuracy of answer (Gagatsis & Shiakalli, 2004), difficulties in understanding problem's aim (Wijaya, et al., 2014), and difficulties in formulating equation from word problem context (Jupri & Drijvers, 2016).

The issue of linguistics difficulty in solving word problems of conic section cannot be separated from the content of the topic itself. Conic sections topic is typically divided into three main parts; i.e. parabola, ellipse, and hyperbola; as mathematical objects resulted from slicing a cone differently (Suparmin & Estikarini, 2014). Each of conic section brings its own characteristics, and automatically, terms used to explain those characteristics. Terms e.g. center, vertex, focus, major axis, minor axis, directrix, eccentricity, etc are used along the content and sometimes differ in meaning and representation according to which conic section being discussed. Thus, incorporating conic section concept into word problem cannot be separated from the ability to make meaning from those terms. Besides, alternating use of symbolic and visual representation of conic sections is always needed to get complete cognitive understanding of the concept (Hitt, 1998). This interplay between symbolic and visual representation should be possessed by students and when it comes to word problem, the challenges brought by conic sections as mathematics topic are intensified.

In regards to those facts, it is imperative to understand and explore the linguistic difficulty in conic sections word problems. Understanding the linguistic difficulty found in students' work may lead to the understanding of helping students facing that difficulty. Thus, the aim of this study is to describe students' language difficulties in word problems with the topic of conic sections.

2 THEORETICAL FRAMEWORK

In order to understand the linguistic difficulties in mathematics context, it is important to understand the linguistic features constructing mathematics word problems. O'Halloran (2005) stated that word problem is constructed by its own language system. This language system organizes choices of language function, mathematical symbol, and visual display. Failure in understanding this system will lead to failure in understanding word problems due to its linguistic features.

Literatures have described several linguistic features or aspects that construct mathematics language. The work of Schleppegrell (2007) and O'Halloran (2010, 2014) explained linguistic features of mathematics using the fusion between mathematics and linguistics. They synthesized three main features of mathematical discourse i.e. multiple semiotic systems, vocabulary, and grammar and syntax. In accordance with them, the work of Lee (2005) highlighted the features of mathematics language specifically for assessment and instructions. The work mentioned the naming power, a power of particular word or phrase to awaken related concepts in mind, besides vocabulary and syntax as the main features of mathematical language. Although these studies categorized mathematical language into three similar features, Lee focused more on the role of word and syntax while the first two did more on multiple semiotic systems feature. Table 1 displays the summary of linguistic features and sub-features of mathematics word problems used to classify cases found in this study.

Table 1. Linguistic features of mathematics word problem

Feature	Sub-features
Multiple semiotic systems	transforming symbol into visual representation transforming visual representation into symbol transforming language into symbol transforming symbol into language transforming language into visual representation transforming visual representation into language
Vocabulary	Same meaning words Math-specific words (technical words) Different meaning words
Grammar & complex syntax	Implicit meaning Dense noun phrases Passive voice Conditional clauses Relative clause

Mathematics text (in this case is word problem) is constructed using language, mathematical images, and symbolism (O'Halloran, 2010). Each of the three has its own system of sign used in meaning-making or in other words, mathematics meaning is created by multiple semiotic systems. The processes happen within this multiple systems are the intertwining transformation process between each pair of the three systems. Not only multiple semiotic systems, mathematics vocabulary also contributes to mathematics language. Lee

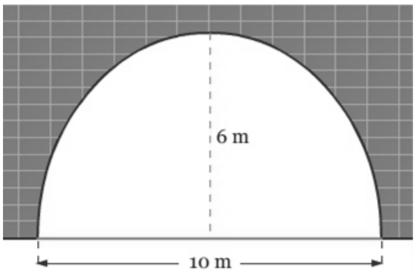
(2005) mentioned the classification of mathematics vocabularies as same-meaning words, math-specific words (technical words), and different meaning words. The way vocabularies are used to construct a text resulting in grammar and syntax feature (O'Halloran, 2014). While vocabularies feature deals with the meaning of a word or term independently regardless its location in a sentence, grammar and syntax feature deals with the structure of words in a sentence and how they create the meaning of the whole text.

3 METHODS

A test consisted of three essay items was given to 28 students of 11-grade. The three items were in the form of word problems with the topic of conic sections. The test items were typical word problems found in the school textbook provided by government (Suparmin & Estikarini, 2014), yet the construction was consulted to a mathematical education expert and students' mathematics teacher regarding the readability and clarity. The test was written in Indonesian language; its English translation is given in Table 2.

The test topic and plan has been announced to students a week before the test. Students could use calculator during the test as the test aim was not to examine students' arithmetic skills, thus it would not affect the analysis of linguistic difficulties. The test was invigilated by the researcher and classroom teacher. During the test, students were also suggested to write any concern they found regarding the questions and testing procedure to make sure that other technicalities would still be considered.

Table 2. Word problems in the test

No	Word problem	No	Word problem
A	A half-ellipse tunnel as given in the picture is located below a bridge. The length and height of the tunnel is 10 meter and 6 meter consecutively. The tunnel will be opened for all kinds of vehicle. Is it possible for a 3-meter-wide and 5-meter-high truck to pass through the tunnel without getting a crash? Explain your steps in getting the answer.  	B	A parabolic solar panel has a diameter (the distance between its edges) of 10 meters and its parabolic section can be modeled as $x^2 = 50y$, *y is the depth of the solar panel.* a. Make an illustration of the solar panel. b. Write a mathematical model to determine the location of solar panel focus.
		C	An elliptical park has 500 meters as its major axis and 300 meters as its minor axis. The park developer wants to make a fountain in each of its focus. a. Make an illustration of the park b. Write a mathematical model to determine the distance between the fountains.

Each answer in student's work was analyzed from the perspective of linguistic difficulties by giving appropriate code(s) of linguistic difficulties features (as displayed in Table 1). The codes were "MSS" (for multiple semiotic systems), "V" (for vocabularies), and "GS" (for grammar and syntax). Then, the codes were recorded and students' answers were classified based on its feature. One answer might have more than one code because of two rationales. First, student's answer might show more than one type of difficulties. Second, as explained before, the linguistic features of mathematics word problems may be considered separated and inter-related at the same time. After being classified into each feature, students' answers in each feature were analyzed by similarities. Similar cases of difficulties were grouped together and the characters of the case were noted down.

4 RESULTS AND DISCUSSION

The findings discussed in this section are explained from the general ones, and then followed by more specific descriptions of linguistic difficulties in each feature. The discussion in each feature is accompanied by example(s) to exhibit clearer description of the difficulty.

There are 28 students participated in the test. Table 3 depicts the numerical data on students' difficulties for each test item. Column 2 shows the average score of 28 students. Student's work was graded based on its accuracy, and each item has maximum score of 10. Column 3 shows the number of cases for each linguistic feature found and column 4 shows the number of cases for type of difficulties found. Because one students' work might have more than one type of difficulty, the numbers do not add up to 28.

Table 3. Numerical data on students' linguistic difficulties for each test item

No.	Average score	Feature		#Difficulties	
A	1.8	MSS	44	transforming visual representation into symbol	23
		Grammar and syntax	11	transforming language into symbol	21
				do not consider implicit meaning	11
B	6.6	MSS	32	transforming symbol into visual representation	9
		Vocabulary	3	transforming language into symbol	9
				transforming symbol into language	12
				transforming language into visual representation	2
				do not understand technical words and/or Indonesian words	3
C	3.9	MSS	24	transforming visual representation into symbol	6
		Vocabulary	1	transforming language into symbol	4
		Grammar and syntax	4	transforming symbol into language	8
				transforming language into visual representation	6
				do not understand technical words and/or Indonesian words	1
				do not consider implicit meaning	4

Note: most found feature is multiple semiotic systems (100), followed by grammar and syntax (14) and vocabulary (4)

Item A was the most difficult to solve with only 1.8 average score out of maximum of 10 points. This interesting fact may be caused by the linguistic difficulties as the item revealed most cases of linguistics difficulties (55) compared to other items. Looking into the item, item A includes picture which is supposedly become supporting information for students to solve the problem, yet that was not the case. Although providing picture as information, the problem needs students to transform the picture, beside the language, into its mathematical symbol; thus the picture itself may be a help to understand the problem, but may not be a help to solve the problem. This finding is similar with what has been found by Gagatsis and Shiakalli (2004). They found that a provided picture of a problem will not necessarily help students to solve the problem, depending on whether the picture itself needs translation process or not. If students have to translate the picture (or transform it to other representations), the picture will make the problem even more difficult to solve.

All three features of linguistics difficulties; i.e. multiple semiotic systems, vocabularies, and grammar and syntax; were found in students' work. For each item, multiple semiotic systems was the most found feature of linguistic difficulties (100 cases for all items), particularly in transforming language into mathematical symbol (34 cases for all items). This means that students' ability in understanding and creating an inter-related meaning among symbol, visual representation, and language was still lacking, particularly from language form into its mathematical expression.

Vocabulary feature was not found as difficulty in item A, while grammar and syntax feature was not found in item B. This probably due to the construction of item A that has only "ellipse" as the technical word, which is actually a common word in conic sections topic. Besides, the important vocabularies given in the item e.g. tunnel; length, height, etc were mostly depicted in the picture given. In item B, the sentences were constructed without using any conditional clause, a clause demanding an effect over a particular cause, unlike the construction of item A and C; resulting in a less complex syntax and less difficult sentences to comprehend. The existence of conditional clause within a text has been found to make the text more difficult to understand (Schleppegrell, 2007).

In general, types of difficulties found in multiple semiotic systems feature were transforming symbol from and into visual representation, transforming language from and into symbol, and transforming language into visual representation. Type of difficulty found in vocabulary feature was difficulty in understanding the meaning of technical word and Indonesian word. Type of difficulty found in grammar and syntax feature was difficulty in considering implicit meaning. Detailed elaboration of difficulties found in each feature will be explained in the following sections.

4.1 Difficulties Due to Multiple Semiotic Systems

In this study, multiple semiotic systems was the most found feature of linguistic difficulties. There were 55 difficulties found i.e. difficulty in transforming symbol into visual representation (9), visual representation into symbol (29), language into symbol (34), symbol into language (20), and language into visual representation

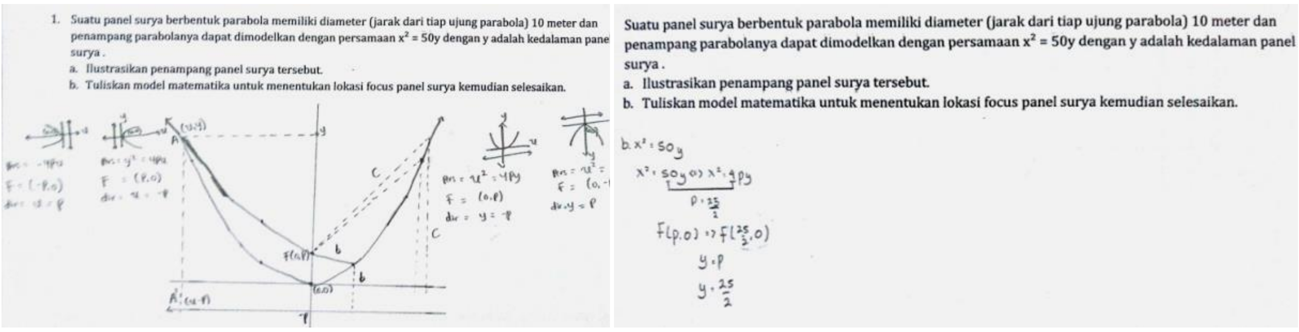
(8). Number of difficulties found in each item is given in Table 4.

Table 4. Number of difficulties in multiple semiotic systems

Type of difficulty	Item A	Item B	Item C
transforming symbol into visual representation	ND	9	ND
transforming visual representation into symbol	23	ND	6
transforming language into symbol	21	9	4
transforming symbol into language	ND	12	8
transforming language into visual representation	ND	2	6

Note: ND means no data of difficulty found for the corresponding type

The difficulty in transforming symbol into visual representation was found in student's work on item B. In this case, item B asked students to illustrate the given parabolic solar panel. Some students did not give any graph of the mentioned parabola, while some others gave a partially correct graph. In the case of transforming symbol into visual representation in item B, the findings showed that not all students who could draw the graph correctly got the correct final answer for the item and the ones who could not, could somehow got the correct answer. This finding may lead to a discussion on how making the graph in item B (i.e. the process of transforming an equation of $x^2 = 50y$ into a graph) is related to determining the focus asked in the second question of the item. Both the process of making the graph and determining the focus can be done by having the symbol of $x^2 = 50y$ alone. This indicated that linguistic difficulties in transforming symbol into visual representation can be addressed as either separated from or inter-related to producing correct answer.



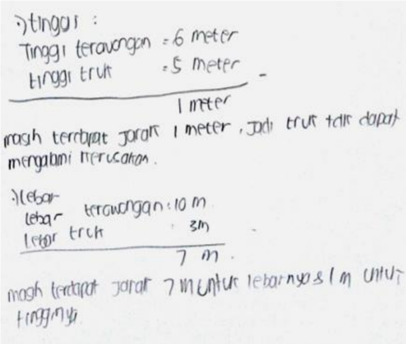
(a) (b)

Figure 1. Examples of difficulties in transforming symbol into visual representation

Representative examples of partially correct graphs are shown in Figure 1. Figure 1a shows a student who could draw a general graph of a parabola with (0,0) as its vertex, yet failed to make use of $x^2 = 50y$ to complete the graph with other elements of the parabola, thus he gave wrong answer to the problem. Interesting fact is shown in Figure 1b in which a student did not make any graph but she successfully figured out the coordinate of the focus using the equation given. This example shows that although a student may fail to produce an equivalent visual representation of a symbol, she/he still could produce a correct coordinate of the focus.

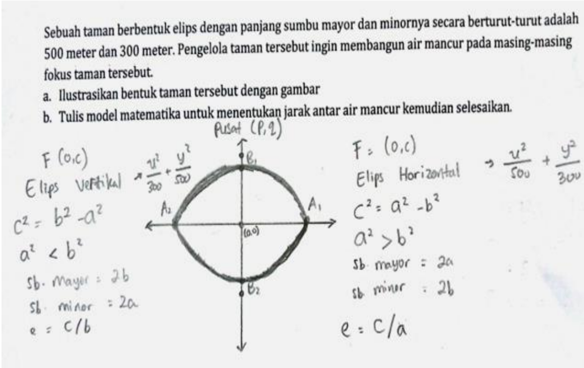
Eisenberg described similar cases in his study where students often treat visual representation of a problem excluded from the problem solving process (2011). He explained that some students prefer to use visual image to solve problem, while others use non-visual analytical processes. However, in mathematics topic involving function, the concern on failure mostly faced by students who prefer non-visual analytical process is emphasized.

The difficulties in transforming visual representation and language into symbol were the most frequent difficulties. In this study, the difficulties were shown through errors in creating correct mathematical expression to represent the given picture. For item A, the difficulties were shown through incorrect use of an arithmetical expression to solve the tunnel problem (shown by Figure 2a). Here, student failed to consider the picture as an elliptical object and use simple subtraction instead (by doing subtraction of $6 - 5$ and $10 - 3$). Difficulty in transforming language into symbol was shown in item C, in which a student indeed used general equation of ellipse, but incorrectly incorporated the major and minor axis into the equation (shown by Figure 2b). The value of the term "major axis" and "minor axis" were incorrectly transformed. Student incorrectly put 500 and 300 as the value of a^2 and b^2, the denominator of $x^2$and y^2, consecutively (as seen in the written expression of $\frac{x^2}{500} + \frac{y^2}{300}$).



Translations:
Height:
tunnel's height = 6 meters
<u>truck's height = 5 meters</u>
= 1 meter
Still has 1 meter so there won't be a crash
Width:
tunnel's width = 10 meters
<u>truck's width = 3 meters</u>
= 7 meters
Still has 7 meters for the width and 1 meter for the height

(a)



(b)

Figure 2. Examples of difficulties in transforming visual representation and language into symbol

Difficulties in transforming symbol into language were found on the process of giving the final conclusion of the answer. For example in Figure 3, student managed to calculate the correct value of $c = 200$ from given information in item C. However, he failed to transform the meaning of c into "the distance between the ellipse center and its focus". This transformation, if done correctly, will lead to the answer of the word problem which is the distance between two foci of the ellipse.

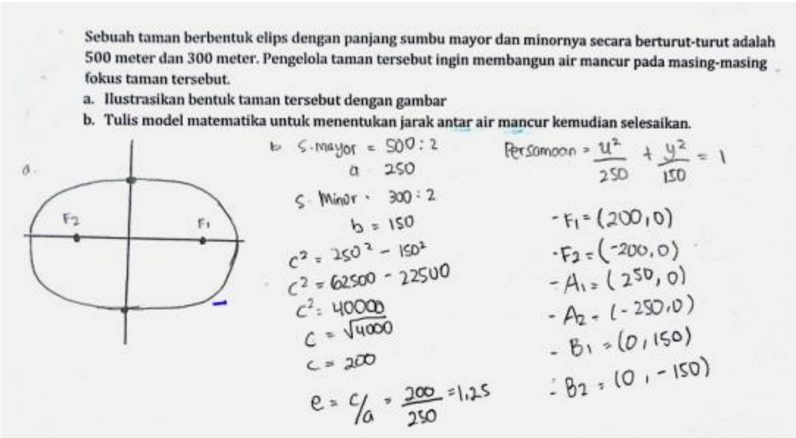


Figure 3. Example of difficulties in transforming symbol into language

The difficulties in transforming language into visual representation were shown through the errors in item C, where the description of a particular park should be put into a graph. The error found was drawing a partially correct ellipse graph e.g. drawing ellipse without pointing out its foci, drawing a general ellipse without incorporating the numerical information given, or incorrectly point out the ellipse elements (axes, foci, vertices, etc). This error could be due to the assumption that the visual representation of the description would be less important compared to the mathematical procedure, as the cases of students directly moved to the mathematical procedure without drawing the graph were also found.

Responding the difficulties in multiple semiotic systems feature, Duval (2006) gave thoughtful opinion for the underlying cause. He explained that the three semiotic systems in mathematical problem are not used to only communicate (as how daily language is typically used), but also to work on procedures within them. That is why no solving process can be done without using, not only understanding, them. Here is where the process of substituting one system with another, called transformation, plays the most crucial role in solving word problem.

4.2 Difficulties Due to Vocabulary

In this study, the difficulties caused by vocabulary feature was not in the form of misunderstanding the "big" vocabularies of conic sections; i.e. ellipse, parabola, and hyperbola; because no student misinterpreted the word "ellipse" as a parabola or a hyperbola, and vice versa. The common difficulties found in this feature were on the understanding of each of conic section elements (e.g. center, axes, foci, vertices, etc). For example, a term “focus” in a parabola means a point where its distance to the points in a parabola will be equal to the distance from those points in a parabola to a given line called directrix. Slightly seen, a focus will be inside the curve of a parabola. In item B, this case may cause the inaccuracy of pointing out the location of a focus (as shown by an example in a circled area in Figure 5). The student correctly determine the coordinate of the focus as (0, 12.5) but failed to put it into the correct location. This action might be caused by accidentally putting the focus on (12.5, 0) instead of (0, 12.5), yet this error could be avoided if student understood that it is not possible for a focus to be outside the parabola curve. An understanding of a technical term in mathematics might be used not only during the process of creating answer but also to avoid an inaccurate answer.

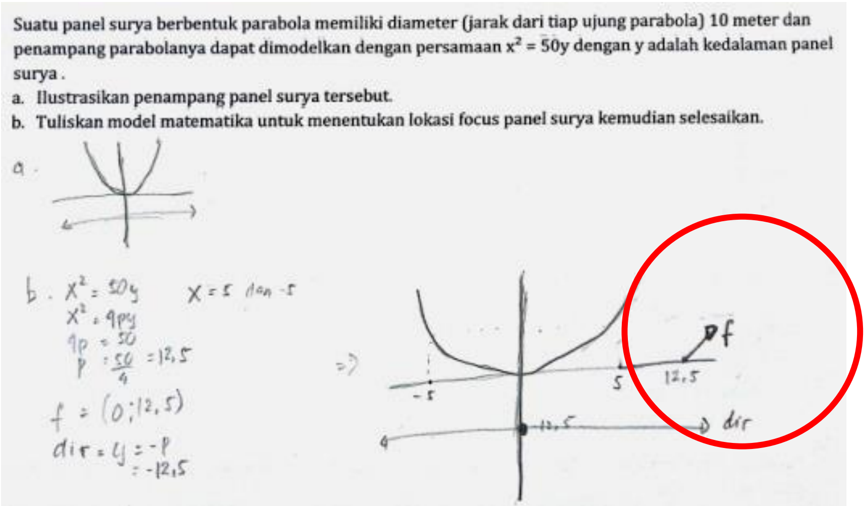


Figure 4. Example of difficulty due to vocabulary

The difficulty caused by not understanding a particular vocabulary was also shown by a student who failed to do item B correctly because he did not know the meaning of the phrase “penampang panel surya” (in this context means a section of a solar panel). He wrote the following respond as his answer.

> *“Apakah yang dimaksud dengan penampang panel surya? Saya tidak mengerti apa itu penampang panel surya. Tapi berdasarkan data persamaan $x^2 = 50y$, maka akan ada bentuk parabola”*
>
> (“What does *penampang panel surya* mean? I don’t understand what is *penampang panel surya*. But based on the data of equation of $\mathrm{x}^2 = 50\mathrm{y}$, then there will be a form of parabola”)

The student managed to figure out that the problem is about a parabola, but because he could not understand the meaning of “penampang panel surya”, he could not work further. In this case, although student’s failure might not be solely caused by his vocabulary difficulty, as O’Halloran (2004) mentioned that the difficulties expand beyond vocabularies, particular vocabulary or term in the text still does contribute to the difficulty of a problem.

4.3 Difficulties Due to Grammar and Syntax

The difficulties caused by grammar and syntax feature were shown when students respond the question without considering the implicit meaning or assumptions agreed within the sentence construction. For example in item B, it does not matter whether the ellipse is vertical or horizontal. In the case of putting an ellipse into Cartesian diagram, outside the context of word problem, indeed the equation of a vertical ellipse is different from a horizontal one. However, in the real context in item B, either horizontal or vertical equation of the ellipse will work. The implicit meaning within the sentence “an elliptical park has 500 meters as its major axis and 300 meters as its minor axis” is that the ellipse may either be horizontal or vertical.

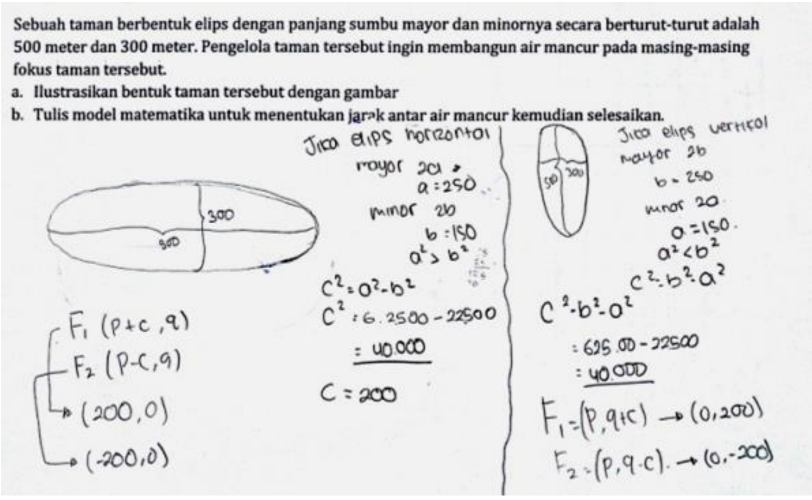


Figure 5. Example of difficulty due to grammar and syntax

Not considering this implicit meaning may lead to confusion or less-effective process as shown by an example in Figure 5. In this case, student did two processes, working on horizontal ellipse (left side) and vertical ellipse (right side). Student managed to find the coordinates of foci for both processes i.e. $(200,0)$ and $(-200,0)$ for horizontal ellipse and $(0,200)$ and $(0,-200)$ for vertical ellipse, but he either could not come into conclusion which ellipse would be used for completing the answer or simply thought that the process was finished. In both cases, student showed difficulty in giving correct answer and it cannot be overlooked that the implicit meaning within the sentence plays a role. Wijaya, et al. (2014) found similar difficulty when students tended to directly jumped to the algorithms without even checking whether it was needed for the context or not. This tendency leaded to the errors in next solving steps and ended in wrong answer. The study recommended much attention on the skill of selecting relevant information and understanding implicit meaning from the word problem given.

5 CONCLUSIONS

There are two concluding points in this article. First, the difficulties due to linguistics feature of mathematics are real and faced by students during word problem solving of conic sections. Second, those difficulties are found in various cases and the features correspond to them (i.e. multiple semiotic systems, vocabulary, and grammar and syntax) and can be seen as separated or related to each other at the same time.

Knowing these facts, it is suggested that teachers should understand word problems not only as a part of a mathematical task, but also as a language object that should be addressed from linguistic perspective. Thus, in order to do so, knowledge of linguistic features of mathematics word problems should be possessed by teachers. By using this knowledge as the underlying guideline, teacher may construct their own way to help students face their linguistic difficulties. Allocating more time and effort to help students exercise their skill in doing transformation among language, symbol, and visual representation may be a useful action. Emphasizing more on the meaning of terms used in conic sections may also be helpful for both teachers and students.

It is not claimed that all possible linguistic difficulties are found and discussed in this study due to its specific mathematics topic and particular word problems given to students. However, this article has tried to address the difficulties of solving word problems from different perspective, linguistic perspective. In the future, the investigation of linguistics difficulties may be broadened by investigating different mathematics topic. The linguistic difficulty on younger students mathematics may also be an interesting issue to study as their language ability may still be developing.

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