

## STUDENTS' MATHEMATICAL ARGUMENTATION IN TRIGONOMETRY

Churun L Maknun<sup>1\*</sup>, Rizky Rosjanuardi<sup>2</sup>, Trisno Ikhwanudin<sup>3</sup>

<sup>1</sup>School of Postgraduate Studies, Universitas Pendidikan Indonesia, INDONESIA

[churun@student.upi.edu](mailto:churun@student.upi.edu)

<sup>2</sup>(Dr.) Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia, INDONESIA

[rizky@upi.edu](mailto:rizky@upi.edu)

<sup>3</sup>(M.Ed.) School of Postgraduate Studies, Universitas Pendidikan Indonesia, INDONESIA

[trisno.ikhwanudin@student.upi.edu](mailto:trisno.ikhwanudin@student.upi.edu)

\*Corresponding author

### Abstract

Trigonometry is a material that many students found difficult. Students should not only tend to lead to the correct procedure or answer, but also how to understand every procedure and applied property. Therefore, it is important to figure out students' mathematical arguments, that is, reasoning that intends to show, explain why a mathematical result is true. The main purpose of this study is to obtain information how students make mathematical arguments in trigonometry. A total of 36 high school students' answers was analyzed to understand their understanding of the trigonometry material through their arguments given in each answer. The students were given three questions related to the trigonometry material they had learned. The questions posed in this study cover solving trigonometry equality, trigonometry identities and application of trigonometry which was represented specifically in sine and cosine law. From the analysis, it was found that most of the students answered correctly on the given problem, but they lacked appropriate argument to explain why it is true. In particular, students failed to understand the periodic system in the trigonometric functions. In this study, it is also found that the algebraic ability is the most common error found in students' answers.

**Keywords:** argumentation in mathematics, high school student, proof, reasoning, trigonometry, trigonometry equality

### 1. INTRODUCTION

Mathematics is a hierarchical subject that prior material needs to be well understood to understanding the next material. It will be difficult to learn mathematics if one lacks to understand a basic material to learn an advanced material. The essence of mathematics essentially departs from the definitions and axioms that then develop (Brousseau, 2002). Understanding in mathematics is not only can do right solution quickly while doing math problems, but also knowing why a procedure or a property is held. Trigonometry is one of the mathematical material that relatively complex and needed to further material. Trigonometry covers some materials, i.e. radiant, sine and cosine definition, sine and cosine law, trigonometry equality, identical trigonometry, graphical trigonometry, application of trigonometry etc. Trigonometry is a prerequisite in the study of calculus such as the indefinite integral of trigonometric functions, limits of trigonometric functions and derivatives of the trigonometric function. Trigonometry also applies to the other of mathematics material such as the three dimensions, vectors and transformation of geometry. So in this case, the lack of understanding of the trigonometry material will affect not only the trigonometry itself, but also affect on the mastery of pre-calculus material which is the basis of more complex calculus materials (Orhun, 2001; Thompson, Carlson and

Silverman, 2007).

The goal of learning mathematics should correspond to the essence of mathematics itself. The students are supposed to have a good ability in generating explanations, formulate questions, and write arguments (National Council of Teachers of Mathematics, 2000). In line with the NCTM report, the National Committee on Mathematical Requirements stated that one of the primary purposes of the teaching of mathematics should be to develop the powers of understanding (Niss, 1970). Thus, a student is not only supposed to accomplish to do mathematics, but also to make arguments and understanding how and why it has the property. Moreover, since students learn these trigonometry concepts for the first time in grade 10th, especially in Indonesia, thus, the importance to engage students to the concept of trigonometry and its property cannot be doubted. Students need to successfully understand the trigonometry to successfully understand other material of mathematics related to.

To know students' understanding of the mathematical concept, particularly in trigonometry, students need to lead to make an argument. The argument is a reason why an idea or statement supported which is produced from a sequence of a process called argumentations (Sriraman and Umland, 2014b). Besides, mathematical argumentation can be defined as reasoning that intends to show or explain why a mathematical result is true. The mathematical result might be a general statement about some class of mathematical objects or it might simply be the solution to a provided mathematical problem. Taken in this sense, a mathematical argument might be a formal or an informal proof, an explanation of how a student comes to make a particular conjecture, reason through a problem to arrive at a solution, or simply a sequence of computations that lead to a numerical result (Sriraman and Umland, 2014a). From the argumentation given by students, it can infer the students' understanding, what they understand, even the misconception which might occur.

Many research had been conducted refer to the students' understanding in a wide range of trigonometry subject areas such as a function of trigonometry and trigonometry identity. Most of them stated that the students' had a lack of understanding in trigonometry (Cavey and Berenson, 2005; Weber, 2005; DeJarnette, 2014; Kamber and Takaci, 2017) and found an error, mistake and misconception in student (Orhun, 2001; Usman and Hussaini, 2017). Those researches mostly focus on the trigonometric functions which refers to the triangle trigonometry, the unit circle of trigonometry and graphical trigonometry with a quite similar type question. This study relatively similar to the previous research regarding the analysis of errors might occur in students' answer. However, this study provided a different type of questions cover analysis of trigonometry equation, validate trigonometry identity and make a general assumption in the application of trigonometry. Previous research showed that the high school students still had a misconception and did not understand the trigonometric functions. Most of them stated that the learning process in a classroom did not facilitate the students to get such an ability. This reason is in line with Brousseau (2002) and Sriraman and Umland (2014b) that classroom learning now focuses on how to learn a lot of math content in a short time without understanding how mathematics is and a lack of developing arguments.

Those findings had been conducted in several countries, particularly in America, Europe and its surrounding areas. However, few researches relate had been conducted in Asia, especially Indonesia in this case of study. Thus, this study will give contribution to the mathematics education areas regarding the student's understanding and mathematical arguments relate to the trigonometry.

## **2.METHOD**

### **2.1 Aims**

This is an exploratory study that analyse students' answers in mathematics problems. The authors emphasize how students argue the given solution of mathematics problem. The topic of mathematics used in this study is trigonometry. A quantitative and qualitative technique conducted in this study. The quantitative data were used to obtain a general score of students' performance in the questions given, despite qualitative data were used to obtain deep information in which case students did well and poorly in mathematical argumentations. From the explanation above, the aims of this study are:

- 1) What is the proportion of students who answered correctly?
- 2) How does students' performance in mathematical argumentations in trigonometry?
- 3) What are the problematic aspects to student in trigonometry?

### **2.2 Participants**

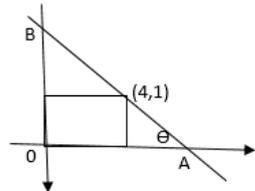
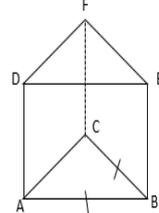
The study was carried out in the second semester of a group of 10th-graders enrolled in a private Islamic boarding school focused on trigonometry concept. Students involved in the study had learned the trigonometric

and they were at the same level of skill. One class to be selected for this research (n=36) contained 11 and 25 of male and female students respectively.

### 2.3 Instrumentation

The instrument used in this study was designed to ascertain the mathematics argumentation of the students on the trigonometry. The problems in this trigonometry were modeled based on the curriculum on trigonometry competency for high school. To obtain a comprehensive data result, an open-ended test type is used to know whether the answer is well structured or less structured (Ennis, 1993). The questions used in this study had been checked for their validity and reliability. The students were given the assessment, told to answer all the question to the best of their ability and to ask any question for any needed clarification about the assessment. Time was not intended to be a factor, however, 80 minutes was allotted for completion. The students were allowed to stop working on the assessment if they felt they could no longer answer the questions. None of the students took no longer than 80 minutes. A scoring guide assigns a certain number of possible points to the appraisal of each step and the solution as a whole. The student also must be proficient in handling reason why it is wrong or right. Reason or response that well explored receives full credit.

Table 1  
 test instrument

<p>1 Evaluate the solution steps below! [with the range <math>0^\circ \leq x \leq 360^\circ</math>] Do the steps are valid? If so please explain. If it does not, write the right solution!</p> $\sin^2 x - \frac{1}{4} = 0.$ $\sin^2 x = \frac{1}{4} \dots\dots\dots(1)$ $\sin x = \frac{1}{2} \dots\dots\dots(2)$ $\sin x = \sin 30^\circ \dots\dots\dots(3)$ $x = \alpha + k.360 \dots\dots\dots(4)$ <p>if <math>k = 0</math>, then <math>x = 30 + 0.360 = 30^\circ \dots\dots\dots(5)</math></p> <p>if <math>k = 1</math>, then <math>x = 30 + 1.360 = 390^\circ</math> (unsatisfied).....(6)</p> <p>Thus the solution of <math>x = 30^\circ \dots\dots\dots(7)</math></p>	
<p>2</p> 	<p>Consider the figure beside!              Is that true that  <math>OA + OB = 5 + \tan \theta + \cot \theta</math>              Why?</p>
<p>3</p> 	<p>Consider the the figure beside!              The length of AB is twice of the length of AC. The length of AD is 40 cm. A student will fill the figure beside with 2 liters of water. What is the shortest length of AC can be made? Give your reason clearly to convince your answer!</p>

Note: This instrument has been translated into english. In this study, students did the test in Indonesia language.

The first question provides an argumentative solution with numbered steps, most of which have specific built-in errors. This question asked the student to evaluate whether right or wrong of the steps in the trigonometric equation solution. The second question asked the student to identify whether a statement right or wrong. This question also asked the student to explain and show the argument regarding the answer they choose. The third question asked the student to considerate a solution of the problem. A good judgment is a need in answering this question.

### 3.RESULTS AND DISCUSSION

#### 3.1 Quantitative Result

The overall score of students presented in the table 2. Most of students did not do well in the test.

Table 2.

Questions	Proportion
1	0.35
2	0.40
3	0.32

Note. Proportion = The number of score compared to the maximum score.

The average of the students' score was poor. Each of the student could only do the task correctly less than 50% of the overall task. Although the questions given were common in students' textbook and daily class assignment, It can be inferred that the students were unfamiliar with this type of question in which they need to make a reason in every step they took. The score obtains from a whole of the process solution and reason, thus, although the students write the right answer at the end of the solution, they would not get an optimum score.

#### 3.2 Qualitative Result

##### 3.2.1 Question Number One

In this question, most of the students can evaluate correctly that the first step is true. However, the students failed to explain why it is true. Of the 36 students, twenty-five students or about 71% of students gave the reason that the first reason was right because the number negative  $\frac{1}{4}$  is moved to the other side thus it became positive  $\frac{1}{4}$ . The notion of moving the number to solve an algebraic form is very attached to the students because the math teacher always uses that concept. Moving the number means the number can be moved to another side in a condition the sign (positive and negative) has to change. However, this concept does not exist in mathematics. In this step, the concept needs to be emphasized is the inverse of the real number. Eight students stated that the first step was wrong. They argued that there is no sinus which has value equal to  $\frac{1}{4}$ . The concept of the trigonometric value is not well understood by the students. It denotes that the student introduced the angle in trigonometry is a special angle only i.e.  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $90^\circ$ . So, the students assume that the trigonometric value of sinus or cosinus is  $0$ ,  $\frac{1}{2}$ ,  $\frac{1}{2}\sqrt{2}$ ,  $\frac{1}{2}\sqrt{3}$  and  $1$  only. They do not know that the range of the trigonometric function is a real number at the interval of  $[-1,1]$ . The rest of the students did not give any reason.

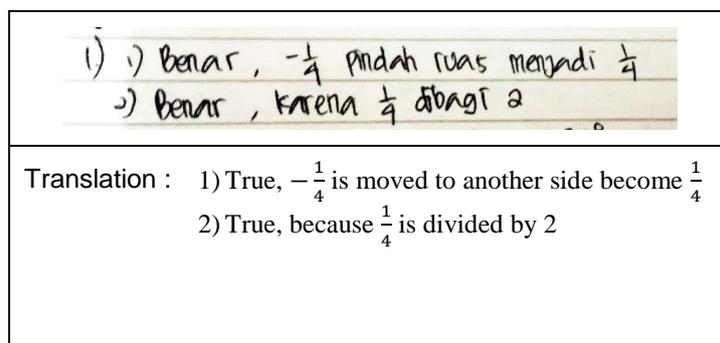


Figure 1. S27's answer in algebraic problem

The second step is the false step because the result should be  $\pm\frac{1}{2}$ . However, 34 of the 36 students answered that the second step was correct. Two students who stated that this step was false did not give any explanation. This indicated that the students did not aware of the properties of an algebra equation. The reasons why they stated this step was true were quite varied. The reasons they gave included algebra and trigonometry. In algebra reasoning, two students gave the correct explanation. They explained that it was true because of the withdrawal the roots of both sides. Fourteen students gave an inappropriate algebraic

explanation (e.g.  $\sin x = \frac{1}{4} \cdot 2 = \frac{1}{2}$ ; because  $\frac{1}{4}$  is divided by 2; the square of  $\frac{1}{4}$  is  $\frac{1}{2}$ ;  $\sin^2 x = \frac{1}{2} + \frac{1}{2} = \frac{1}{4}$ ). In trigonometry reasoning, five students gave the reason that it was true because there is a trigonometric value equal to  $\frac{1}{2}$ , that is, when  $x = 30^\circ$ . One student stated that "in table (table of the value of special angle)  $\sin x = \sin 30 = \sin \frac{1}{2}$ ". It is an interesting finding that the students think that it will always be true when they meet a value corresponded with the special angle in trigonometry. The fact that there are some students who write " $\sin 30 = \sin \frac{1}{2}$ ". The rest of the students did not give any reasons.

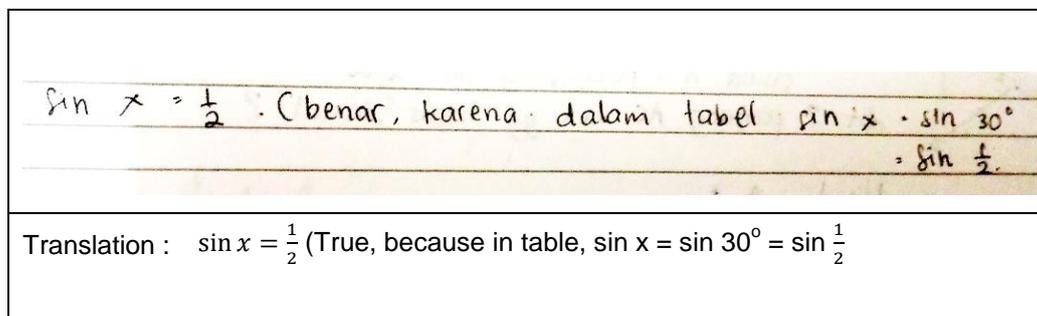


Figure 2. S5 student's answer (explanation in step 2)

The third step is a partially true step because the second step is false, thus the third step cannot be right completely. All of the students stated that this step was true, twenty-one of twenty-nine students who gave the explanation stated that this step was true because of  $\sin 30^\circ = \frac{1}{2}$ . Although this reason is correct, the authors expected that the students would give further explanation, like why they intend to write  $\sin 30^\circ$  instead of  $\cos 60^\circ$  knowing the fact that both of these have the same value, that is  $\frac{1}{2}$  and why they choose an acute angle.

Eight students explained this step was true because of  $\sin \frac{1}{2} = \sin 30^\circ$ . This wrong equality appears again in students' answer. This indicated a misconception in students' understanding about trigonometry which students may have from previous inadequate teaching, informal thinking, or poor remembrance (Allen, 2007). The rest of the students did not give any explanations.

In the fourth step, one student answered that the fourth step was false, i.e. wrong because the alpha is unknown and the rest of the students stated that this step was true. Thirteen students explained that it was true because of the formula. Five students explained their answer by state the notion if function (e.g. true, because we will determine the function of  $x \rightarrow x = \alpha + k \cdot 360^\circ$ ). Four students explained that it was true because in accordance with the universal set. Eleven students did not give any explanation. Step fifth and sixth are similar. In these steps, twenty-three students did not give an explanation why  $390^\circ$  is unsatisfied and why  $30^\circ$  satisfied. Only seven students who gave a correct answer why  $390^\circ$  is unsatisfied and why  $30^\circ$  satisfied (e.g. because in the question, we have ). The rest of the student did not give an appropriate explanation (e.g. true, because  $390^\circ$  does not satisfy sinus properties). In the last step, the students need to make a conclusion refer to the steps. All of the students but one student (S28) did not give any explanation. However, the explanation provided by S28 is not relevant (e.g. true, because  $390 - 360 = 30$ ). Generally, students know why  $30^\circ$  is a solution set and why  $390^\circ$  does not. All of the students totally agree with the solution process which indicates that the student failed to recognize the false solution step.

### 3.2.2. Question Number Two

Twelve students able to answered correctly and showed how it should be without explaining it in detail why every step was obtained, e.g. they only wrote that the length of  $CA = \frac{1}{\tan \theta}$ . Surprisingly, twenty students started the wrong process, but they end up with the correct solution. The wrong processes they took to get the solution were varied (e.g. could not read the coordinate correctly, wildly wrote the solution process in order to get the right conclusion and state the unlogical conclusion). The proof neither referred to the question, nor the conclusion. Two students tried to prove the statement by measuring the length of a triangle, then calculate the opposite over adjacent (Figure 4). This solution came up because the students saw the question correspond to the tangent and cotangent. Since AOB is a right triangle so they performed that property. It cannot understand why they can provide 3,3 in the length of the triangle side.

2.)

$OA + OB = 5 + \tan \theta + \cot \theta \rightarrow \text{dalah}$

$OA = OC + CA = 4 + \frac{1}{\tan} = 4 + \cot \theta$

$OB = OD + DB = 4 + \frac{1}{\cot} = 1 + \tan \theta$

$OA = OC + CA = 5 + 4 \tan \theta \rightarrow \text{dalah seharusnya } \cot \theta$

Translation:

$OA + OB = 5 + \tan \theta + \cot \theta \rightarrow \text{false}$

$OA = OC + CA = 4 + \frac{1}{\tan} = 4 + \cot \theta$

$OB = OD + DB = 4 + \frac{1}{\cot} = 1 + \tan \theta$

$OA = OC + CA = 5 + 4 \tan \theta \rightarrow \text{false it should be } \cot \theta$

Figure 3. Student's answer in question number two

2.)

(Solusi) ~~Solusi~~ ~~Bener!~~ ~~Salah~~

$(4,1)$  kon bukti :  $OA + OB = \frac{a}{b} + \frac{b}{a} = \text{Jwb: } \frac{3.3}{3.3} + \frac{3.3}{3.3} = 7$

$OA + OB = 3.3 + 3.3 = 6.6$

Translation:

(Wrong)

Because, proof :  $OA + OB = \frac{a}{b} + \frac{b}{a}$ . So  $\frac{3.3}{3.3} + \frac{3.3}{3.3}$

$OA + OB = 3.3 + 3.3 = 6.6$

Figure 4. Student's answer in question number two

Seventeen students ignore the angle of sinus, cosine, and tangent (e.g. only write  $\frac{1}{\tan}$ , without angle right after the "tan"). This writing appeared several times in students answer inferred that they tend to omit the angle. This finding corresponds to the question one when the students' equalized between  $\frac{1}{2}$  and  $30^\circ$ . Based on the analysis, they correctly answered the posed question. However, they failed to justify every step they took, what property they used, and why they used it (e.g. it is false, because of  $\theta = 4 \tan \theta$ ). Thus, knowing the right answer in mathematics without knowing the reason behind all of the properties used leads to lack of understanding. Fourteen of students did not make a conclusion regarding the question posed. Although they provided the correct answer they did not review what was asked the question. One student did not answer this question.

### 3.2.2 Question Number Three

All of the students had same approach to solving this question. This approach is not totally wrong, however the author expected that the students could find the numerical length solution of AC. Seventeen of the students who answered, gave the logic step in solution in getting the answer. The analysis found that seven students did the same mistake as question 1 and question 2, that is, They wrote  $\sin \frac{1}{2}$  (Figure 4). They could not distinguish between which one is the solution of function sine and which one is the domain angle. Moreover, the illogical step happens again in the third question as before happens in question two, as many as 16 students wrote the wrong step, sometimes wrong number, but the end up in the same solution as the students who answered correctly from the start until the conclusion. As many as seven students without any consideration change the mathematical symbol “more that” to “less than” then change it again to “more than” easily (Figure 4). It can be inferred that the students do not get an understanding in mathematics symbol that every mathematics symbol has a particular meaning, use and property.

$V_{prisma} = L \cdot \text{alas} \cdot t$   
 $L \cdot \text{alas} A = \frac{1}{2} \cdot a \cdot t$   
 $= \frac{1}{2} \cdot AB \cdot AC \sin A$   
 $= \frac{1}{2} \cdot AC \cdot AC \sin A \cdot 4$   
 $= 4 AC^2 \sin A > 2$   
 $\Delta AC^2 = \sin A < 2$   
 $AC^2 = \sin \frac{1}{2}$   
 $AC^2 > \frac{1}{2} \sin A$   
 $AC^2 > \sqrt{\frac{1}{2}} \sin A$   
 $AC > \sqrt{2} \sin A$

Alasan : salah karena tdk sesuai rumus

Translation : Reason: False, because it is not match with the formula

Figure 4. S12's answer in question three

From the answer of S12, He wrote that the reason regarding this question is false because the solution is not match with the formula. Although he tried to provide the reason in this question, however, the reason given is not what author expected and it is an inappropriate reason. Instead of providing the explanation of the step solution, he provides the reason of his answer.

## 4 CONCLUSION

From the result above can be concluded that students generally have neither understanding nor solving the trigonometry questions given. The average score they got was very low, less than or equal to 40 of 100. This happens can be caused by two aspects. First, the student has no proper prior knowledge, particularly in algebra operation and its properties. It can be shown from the students' answer while doing the question that they still make an error when they have to find the solution of  $x$  in the square root. Instead of to withdraw the square root the student divided by a specific number. Second, the students have very few understanding of trigonometry, particularly in trigonometry properties. It can be inferred from the students' answer that most of them find difficult to explain, reason and make an argument regarding the trigonometry statement. Because of they cannot explain correctly, then the score they get is low.

From the qualitative result, generally, the students' performance in mathematical argumentations in trigonometry is mostly inappropriate. Although the students give the reason as the question asked, however, the reason does not have a connection between the steps. For example in question number one, instead of making an argument in a particular step to the previous one—e.g. the third step exists because the second step, what to do or what concepts should be raised to be able to bring up the third step? — the student writes the argument solely in each step make the reason provided is not logically understand. From the analysis from question one to three, It can be inferred that it is difficult for students to learn to consider, evaluate, and build on the argument in the mathematical statement because they are still developing their own mathematical understandings. They still do not understand the definition of a trigonometric value, cannot differ between angle and a value of the trigonometric function, unable to construct the causes and consequences. Moreover, many students ignore mathematical symbol and do not engage with it. They can easily change the mathematical symbol without put on the properties tied.

To sum up, These results suggest that there are many things in teaching and learning of mathematics that need to be fixed particularly in trigonometry. For the teacher of mathematics, knowing the students' weaknesses in mathematics, i.e. in which part of the material need to emphasize or what student ability need to enhance is useful to engage teaching and learning process.

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