# UPPER ELEMENTARY GRADES STUDENTS' ALGEBRAIC THINKING ABILITY IN INDONESIA 

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#### Abstract

This research comes from an idea that algebraic thinking is one of the important things in learning mathematics for elementary school students. Algebraic thinking ability is a transition process from arithmetic learning to algebraic learning. Students in elementary school must have this ability as prerequisite for advanced mathematics (junior high school). Besides that, the Algebraic Thinking ability required students in solving mathematical problems. The main purpose of this research is to describe students' algebraic thinking ability based on algebraic thinking characteristics, knowing the learning trajectory of learning barriers to overcome these obstacles. Therefore, conducted a research using descriptive research method. Data collection using test, interview and questionnaire techniques to students in upper grades of elementary school which located in urban areas, mid areas, also transitional region. Based on the analysis of the results of algebraic thinking ability instrument test from 78 students, comparing with the indicators of algebraic thinking in upper grades of elementary school in Indonesia, it shows that algebraic thinking skills of upper elementary grades in Indonesia are classified as less good. In addition, researchers classify the barriers of algebraic thinking into three types, ontogenic obstacle, didactic obstacle, and epistemological obstacle, therefore with the data generated a learning trajectory to overcome the obstacle of algebraic thinking. Learning trajectory as a result of this study will be developed as algebraic thinking design learning.


Keywords: Algebraic thinking, descriptive, learning obstacles, mathematics.

## 1. INTRODUCTION

Algebraic thinking belongs to one of the most important mathematical problems in mathematical problemsolving. Algebraic thinking students' ability in Indonesia is still low. Based on the results of the PISA (Program for International for Student Assessment) survey in 2012, the mathematical ability of Indonesian students is at 64th position from 65 countries surveyed (OECD, 2014, p. 5). It is the same learning process that is not the student has not been maximized so as to experience the algebra thinking. Brousseau (2002, p. 10) classifies didactic obstacles into three kinds of ontogenic, didactic, and epistemological obstacles. Ontogenic obstacles are barriers that occur due to limited or limited neurophysiological factors. While
didactic obstacles are barriers derived from a didactic situation. Learning that does not pay attention to the demands (learning trajectory) which is viewed from the stage of thinking can be a barrier in learning. As well as the obstacle epistemological, this obstacle comes from the notion of a person who is limited to certain concepts and difficulties when faced with different conditions with the same content.

The low ability of algebraic thinking of the students is hampered in the representation of problems and ability in making strategies to solve mathematical problems. For example, students have difficulty in describing and analyzing numerical patterns because students do not understand the problem to be solved in a pattern and do not know how to determine the next pattern as a solution in solving math problems. This is because less habituation in learning to solve mathematical problems with the form of numerical and geometric patterns. Another cause, because of the weak priority of students to solve the problem solving math problems. In addition to doing a modeling in mathematical problems, other factors are the deficiency of ability to do the problem situation modeling with objects and use representations, such as graphs, tables, and equations to draw conclusions.

Blanton \& Kaput (2005, p. 7) explain that algebraic thinking can be seen as a process in which the generalizations of mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age- appropiate ways. Algebraic Thinking as a transition from arithmetic to algebra has some focus should be noticed. First, in Algebraic Thinking students focus on the relationship of one element with other elements, not just numerical answers; Focus on operation and inverse; Focus on causal link in solving a problem rather than solving problems without understanding meaning; Focus on numbers and letters instead of numbers; and a thorough focus on the meaning of the sign indicating equality or a relationship. These five adjustments of focus are arithmetical domains, but they also represent a shift in the idea of fundamental development for algebraic study (Cai and Knuth, 2005, p. 5). In addition, algebraic equations can offer many solutions. It gives students the opportunity to determine effective solutions and solutions that are different from the usual. This view is an important role played by teachers, in helping students to improve algebraic thinking (Barbosa \& Borralho, 2011, p. 3).
Algebraic thinking is is important to be accustomed since elementary school, Kilpatrick, et al. (2001, p. 8) argues that just as the elementary and middle school mathematics curriculum should prepare students for the study of algebra, so it should also include attention to other domains of mathematics. Algebraic Thinking is a support skill for students to learn formal algebra in the next level. It should get special attention because algebra is one of the materials that is quite difficult for high school students. Several studies have shown that high school students have difficulty in learning concepts and operations of algebraic calculations (Hidayati, 2010, Marsetyorini and Murwaningtyas, 2012; Herutomo and Saputro, 2014; Permatasari, et al., 2015). In addition, the transition from arithmetic to algebran has been proved to be difficult for students, and it is now widely recognized that students need an earlier opportunity to engage in algebraic reasoning in schools (NCTM, 2000 p .161 ). Some of the results of the research study confirm that algebraic thinking should be accustomed since primary school. Having an algebraic thinking base since elementary school, students are expected to reduce the difficulty in learning algebra formally.
Development of Algebraic Learning Algebra Learning in Different Countries. Researches conducted in America (Moyer, Huiker, \& Cai, 2004), Singapore (Fong, 2004), Russia (Schmittau, 2004), Korea (Lew, 2004), China (Cai, 2004), Australia (Booker \& Windsor, 2010, p.2) shows that the curriculum in primary schools has several different ways of introducing algebraic thinking. It is also clarified by Cai who states that most of the school mathematics curriculum separates arithmetic and algebra. Arithmetic became the primary focus of elementary and algebra mathematics as the main focus of high school and upper secondary mathematics. This separation makes it more difficult for students to learn algebra in the middle class (2004, p. 2-3). The results of Booker and Windsor (2010, pp, 7) research on seven-year-old primary school students found that making representations and solving problems with solutions that students acquired themselves in various ways was a step in preparing algebraic thinking. Students develop their algebraic thinking by making generalizations of the solutions they get. The Mestre \& Oliviera (2012, p. 5) Study shows that grade 4 students have begun to develop algebraic thinking. The development of algebraic thinking begins by revealing the generalization of numerical relationships in various representations. Radford's research (2010: 2011, 2012) found that non-symbolic algebraic thinking has been shown by the student at the age of 7-8 years. Based on the results of some of these studies, it is probable that algebraic thinking can be developed in mathematics learning from elementary school.

The scope of algebraic thinking in primary schools includes generalizations based on patterns, facts, phenomena or existing data, solving problems and communicating ideas through symbols, tables, diagrams, or other media to clarify circumstances or problems (Alghtani and Abdulhamied, 2010, p.3) . Meanwhile, according to NCTM (2000, p. 158), algebra thinking of elementary school students developed into the following various indicators:

Table 1. Algebraic Thinking's Indicators

| Instructional programs <br> prekindergarten - grade 12 | Grades 3-5 |
| :--- | :--- |
| Understand patterns, relations, <br> and functions | Describe, extend, and make generalizations about geometric and <br> numeric patterns |
|  | Represent and analyze patterns and functions, using words, tables, <br> and graphs |
| Represent and analyze <br> mathematical situations and <br> structures using algebraic <br> symbols | Identify such properties as commutativity, associativity, and <br> distributivity and use them to compute with whole numbers; |
|  | Represent the idea of a variable as an unknown quantity using a <br> letter or a symbol |
|  | Express mathematical relationships using equations. |
| Uepresent and understand <br> quantitative relationships | Model problem situations with objects and use representations such <br> as graphs, tables, and equations to draw conclusions |
| Analyze change in various <br> contexts | Investigate how a change in one variable relates to a change in a <br> second variable |
|  | Identify and describe situations with constant or varying rates of <br> change and compare them |

The development of algebraic thinking skills is greatly influenced by the activity of learning mathematics in the class. Teachers play a role in determining learning activities in the classroom that can facilitate students in developing algebraic thinking skills. The learning process consisting of preparation, implementation and evaluation must be well prepared. Through design based on research it is an alternative to develop a design of students' algebraic abilities in primary school through a process of didactic situation analysis, non-taped analysis and retrospective analysis conducted cyclically.
Suryadi (2008, p. 6) states that teaching materials should be designed in such a way that students find concepts, procedures, or principles not directly but through a series of activities. Therefore, teachers should be able to present didactic activities that facilitate students to develop algebraic thinking and overcome student learning barriers. The learning barriers that students experience in learning can be predicted when teachers design the lessons to be presented. By analyzing the learning obstacles facing students, teachers can present doactic designs in the form of Hypothetical Learning Trajectory (HLT) that can help students to develop algebraic thinking according to the instructional sequences presented by the teacher. Therefore, descriptive research is done with the title of upper elementary grades students' algebraic thinking in Indonesia. The purpose of this research is to describe algebraic thinking ability of upper elemtary grades students based on algebraic thinking characteristic, to know the factors causing algebraic thinking barrier and make learning trajectory to overcome the obstacle.

## 2. RESEARCH METHODOLOGY

Descriptive research is intended to describe a specific situation or population area that is factual in a systematic and accurate. Descriptive studies are tools for finding new meanings, explaining conditions of existence, determining the frequency of occurrence of things, and categorizing information (Danim, 2002, p. 41). This research will describe algebra thinking process of learning of upper elementary grades students and algebra thinking characteristic of upper elementary grades students. In educational research, Fraenkel at all (2011, p.15) says that the most common descriptive methodology is the survey, as when the researchers summarize the characteristic (individual abilities, preferences, behaviors, annnnnd) of individual groups or sometimes physical environments as school).

Research by determining problems and preliminary studies in the form of literature studies and discussions with various parties to obtain preliminary information on the issues to be studied. The next step has been to formulate the research problem in the form of questions that answers should be sought through research and formulate the research problem, then have chosen the approaches, the research method in accordance with the formulation of research problems, have determined the data source, has chosen data collection
techniques, and data analysis techniques. Data collection was done after the instrument is completed. The data has been obtained is analyzed and the conclusions are drawn.

The participants of the study were selected using purposive random sampling technique. Sugiyono (2013, p. 53) states "purposive random sampling is a technique of collecting data sources with certain considerations". Researchers have selected sample data sources by considering the focus of the research, so that the sample data sources in this study are teachers and students of elementary school.
Data collection techniques that have been used in this study is triangulation in the form of tests, interviews and questionnaires. In this study the research is the main instrument. Researchers as the main instrument role to set the focus of research, determine the source data, collect data, analyze data, and conclude data based pemahamnnya. But researchers are also using auxiliary instruments to collect data.
The test instrument has been validated by the expert and elementary school teacher, besides the researcher distributes the questionnaire after the process of questioning to the students. The results from the questionnaire stated that some students said they could answer the given questions. Students understand the problems and they have found the problems in their daily life. Students also said that they never get a problems related to algebraic thinking from their teachers in class IV-VI. Most students feel the time provided is enough to do the problem and believe the answers given are correct. However, there were some students face difficulty in determining the purpose of the given problem, the student must read many times, ask the teacher about the purpose of questions. This is because the ability of problems representation in some students is still low. On the other hand, the given problem have been validated by the team of experts and teachers as basic, with the validation that the problem of algebraic thinking given is good and varied.

The data analysis technique used in this research refers to the technique proposed by Miles and Huberman (in Sugiyono, 2013, pp. 91-99) which consists of: (1) data reduction; (2) data presentation (data display); (3) conclusion drawing.

## 3. RESEARCH RESULT AND DISCUSSION

The students' algebraic thinking tests and interviews were conducted on algebra learning upper elementary grades students. The tests were conducted on 78 students from three different elementary schools, namely SDN Banjarsari, SDN 184 Gegerkalong Girang, SDN 085 Cimbuleuit.

### 3.1 Findings

### 3.1.1 Learning Process

Based on the results of interviews conducted on upper elementary grades teachers, it can be seen that the ability of students about the pattern of mathematics in the form of numbers and plane is still lacking. Some obstacles found in learning about patterns are the lack of basic ability in logic that makes students face difficulty in reasoning. In addition, students experience technical problems in doing the problem such as accuracy, and carelessness in the process of making the problem. Thus, the lack of students' skills on the patterns impacts the ability to analyze patterns and present them in other forms (see table 2). On the other hand, most students have mastered the skill in using the "=" symbol to apply relationships between varying numbers and predictive set. However, during the learning process it is still found obstacles in understanding the problem. Students should be assisted in representing the problem so that it is understood that the solution of the problem with the use of the symbol " $=$ ". The students' ability regarding representation, such as graphs, tables, modeling etc., is still very less. This is because the lack of habituation in using graphs and tables in learning. In addition there are some suggestions in learning algebraiac thinking, should be accustomed since lower elementary grades, so that when they are in upper elementary grades they have already accustomed to algebraic thinking activities. Furthermore, when they go into junior high school, they have already had good skiil in formal algebra.

Learning algebraic thinking in upper elementary grades is not included in the curriculum, but it is implied in some basic competencies that use algebraic thinking processes. The following basic competencies are:

Table 2. Basic Competence of Algebraic Thinking

| Characteristics of Thinking | Characteristics of Thinking |
| :---: | :---: |
| Describing, extending, and making generalizations about geometric and numeric patterns; | 4.11 Predicting the pattern of plane and space sequence by using concrete models |
| Representing and analyzing patterns and functions, using words, tables, and graphs. | 4.8 Identifying space ithat constructed from another spaces, also determining its surface area and volume |
| Expressing mathematical relationships using equations. | 4.3 Solving the problem about predicting the result of addition, substitution, multiplication and devision of counting number and fractions |
| Modeling problem situations with objects and using representations such as graphs, tables, and equations to draw conclusions | 3.3 Explaining and making doing predicting the result of addition, substitution, multiplication and devision of counting number and fractions |
|  | 4.3 Solving problems which related to multiplication and division of fractions |
|  | 4.10 Organizing and presenting data which related to students it self and comparing it with data from the surrounding environment in form of table, picture diagram (pictogram), bar charts, or line charts |

### 3.1.2 Results of the Algebraic Thinking Test of Upper Elementary Grade Students

The research findings indicate that there are students who are able and unable to work on the algebraic thinking test instrument. After doing the data field analysis the data of upper elementary grades students' algebraic thinking are as follows:

Table 3. Results of the Algebraic Thinking Test

| Problem Indicators | No. <br> Problem | amount <br> Correct answer | Percentage <br> $(\%)$ |
| :--- | :---: | :---: | :---: |
| Finding a plane pattern | 1 | 41 | $53 \%$ |
| Create a table based on pattern | 2 | 39 | $50 \%$ |
| Resolving open-ended problems about the appraisal of <br> the sum, difference, counting number | 3 | 59 | $76 \%$ |
| Completing the breakdown of the chest count <br> assessment | 4 | 40 | $51 \%$ |
| Solving <br> multiplication ofmultiplication and division of fractions |  | 44 | $56 \%$ |
| Creating charts and organize table data. | 5 | 37 | $47 \%$ |
| Average | 6 |  | $55.5 \%$ |

From the table above, it can be seen that $53 \%$ of students can describe, convey, and generalize about geometric and numerical patterns. Problems that can be done by students in finding patterns and counting the number of triangles measure the ability in describing, conveying and making a generalization of geometric patterns. Through the question students are required to perform an activity of thinking, and to search using their own strategy, both in real and imaginative about a pattern that is right to find the answer on the matter. After students find the right pattern then the student will look for small triangle number as the smallest element in the pattern according to the order in question. At the time of the process of looking for the right pattern occur the examples in the minds of students called algebra thinking.

Meanwhile, the number of students who are able to show and analyze patterns and functions, using words, tables, and graphs is about $50 \%$. Student algebra thinking process is shown by the activity of finding a pattern of cube tower by counting its surface, then the students record the result of pattern analysis into the table. Recording process of the data found into the table is an algebraic thinking activity that occurs in students.

The number of students who can express mathematical relationships using the same marks occupies the highest percentage of $76 \%$. In the process of solving the open question of a counting operation, the student will try to look for possible answers. The possibilities that the test student takes into consideration the meaning of the symbols contained in the counting operation. Thus students experience algebraic thinking processes in search of possible answers to find the right answers.
In algebraic thinking indicators model the problem situation with objects and use representations, such as graphs, tables, and equations to draw conclusions, about $51 \%$ of students are able to do so. Problems developed in this inductor include the problem-solving problem of chopping, problem-solving of multiplication and or multiplication of fractions, charting and organizing table data. The process of working on the problemsolving problem, the students interpret the existing in the problem into the form of numbers. Through this process the students are able to know the equations logically and determine the answers to the questions given. Meanwhile, on solving the problem of multiplication and or multiplication of fractions, students solve it by using a proper modeling with the situation in question. After doing a modeling the student will know a value in the same unit then find the right answer. Meanwhile on the problem of making graphs and organizing table data, students present the same data in different forms. The process in the presentation is algebraic thinking skills.
Therefore, it can be known that the average algebra thinking ability of the students is $55 \%$. The average grades obtained by all students are classified into five categories. According Purwanto (2006, p. 82) the calcification is:

Table 4.Category of Student Values

| Value | Value |
| :---: | :---: |
| $90-100 \%$ | Very good |
| $80-89 \%$ | Good |
| $65-79 \%$ | Enough |
| $55-64 \%$ | Less good |
| $<55 \%$ | Less |

It can be concluded that based on algebraic thinking indicators, the algebraic thinking ability of upper elemntar grades is less good.

### 3.2 Discussion

### 3.2.1. The Upper Elementary Grade Students' Algebraic Thinking Ability

Generally, the development of elementary school mathematics curriculum is directed to improve students' life skills. Mathematics learning is aimed at students having the competence of spiritual attitudes, social attitudes, knowledge, and skills. Algebraic thinking is one of the mathematical skills that must be mastered by elementary school students. Algebraic skills are explicitly contained in some basic competencies in the upper elementary class. The learning includes about the concept of pattern analyzing patterns and presenting in other forms (tables), using symbols "=", and graphical representations, tables, modeling etc.

In general, the pre-algebra learning process in the baseline consists of opening, content and cover. The implementation of pre-algebra learning is adapted to the right method, but there are still some obstacles. These constraints come from students and teachers. Students are constrained by the knowledge of natural preconditions of algebraic thought that is lacking; it is due to lack of habituation in using graphs and tables in learning. While teachers are constrained from the delivery of coherent material. This is because students are not accustomed to think algebra since low grade elementary school, so there are steps that are missed to build algebraic thinking competence.

### 3.2.2. Algebraic Thinking' Obstacles

Learning obstacle can be used by teacher for designing algebraic thinking learning. Kinds of learning obstacles are ontogeny obstacle, didactical obstacles obstacle that caused by student's inability or limitedness neurophysiologist. This genetic factor makes students difficult to explain the meaning of something. So they will have difficulty in connecting and learning new condition in their daily life.

Didactical obstacle comes from didactical situation. This obstacle dependent to its educational system. So it is very connected to term that known as "learning trajectory". While learning trajectory is forbidden, it will become a big problem for students to understand learning. Besides that, the complexity of subject matter also effect the success of learning activity. It should be arranged from simple part to complex one.

Epistemological obstacle (Brouseau in Suryadi, 2010, p. 10) fundamentally is people's knowledge for certain context. If the people face to other context, they will be confuse, difficult or enable to use their knowledge
While some wrong answers can be seen in the picture below:


The answer of Figure 1 shows that student know number of triangles in one pattern but do not understand for next pattern. In Figure 2, student's answer is not accurate in determining the number of cube surfaces. As for Figure 3, student can determine number operation correctly, but it is uncorrect for context of the problem as shown. Problem here shows meaningful rectangle and triangle plane. In Figure 4, student confuse to determine the value of each plane so the answer is uncorrect. Furthermore, in Figure 5 students can not determine the weight of each piece of fish, so difficult to find the total weight of fish. In Figure 6 student do mistake in making graphs in accordance with the data given. To explain days $0-4$ in x-axis, student false in determining position of point 0 and has difficuly in determining its coordinates.

Beside using test instrument, researchers conducted interviews on several students with attention to high, middle and lower student abilities. following some interview conversations.
Q : How did you answer the question number 1 ?
S2 : By summing the mount of triangles, then adding and multiplying by 4
Q : How did you answer the question number 2?
S2 : By calculating number of cubes then adding it
Q : How did you answer the question number 3 ?
S2 : By looking for a number that when it adding each other will produce 9 , also for 8 , and I find $4+4$
Q : How did you solve the problem number 4?
S2 : Elephant and donkey group, because I assume that elephants are heavier and stronger.
Q : How did you solve the problem number 5?
S2: By calculate it that $1 / 3 \times 300,1 / 4 \times 300$ Then summing all of its parts.
Q : How did you answer the question number 6 ?
S2 : Make the graph is easy
Meanwhile, the example of interview transcript 2 shows the students who, as a whole, has not been able to do algrebraic thinking well It can be seen from the question no 1, the students is able to show a small triangle in a pattern, but he has not known the strategy used to find out the following pattern. The same thing happens in question 2, the students know that there is a change in the number of surfaces on the cube tower, but he can not determine the number of surfaces in the existing cube towers, and the cube tower 5 . In question no 3 , the student knows that he must find the right number in the counting operation on the question, but he does not ignore the symbols of rectangles and triangles that have their own meaning. Besides, the student also can not explain the rationale reason in answer no 4, the student replied that he saw from the nature that elephant has a great strength so that elephant group and four donkey are able to win the tug of war. Similarly to questions no 5 and 6 , in question no 5 the student cannot correctly interpret the question and he assume that the overall weight of the fish is 300 grams , so they look for the head and tail of the fish by multiplying the fractions by 300 gram . While on the question no 6 of 6 can not explain the meaning of the graph he made, other than that there was a mistake in placing the zero in the cartesius line.
Based on the above description, the obstacles of algebraic thinking that occur in upper elementary grades are obstacles in describing, conveying, and generalizing about geometric and numerical patterns, and barriers in showing and analyzing patterns and functions, using words, tables, and graphs, obstacles in showing mathematical relationships by using equal signals, as well as obstacles in modeling problem situations with objects and using representations, such as graphs, tables, and equations to draw conclusions.

### 3.2.3 Learning Trajectory

Kansanen (in Suryadi, 2008, p.s 7) argued that relation between teacher - content - student represent in the figure of didactical triangle. The relation between teacher and student called as Pedagogical Relation (PR) while the relation between student ant content called as Didactical Relation (DR) and the relation between student and contennt can be anticipated by making its learning trajectory. Learning trajectory has function to know the arrange of subject matter that should be explained so it can lead student' development systematically. Learning trajectory include prerequisite knowledge for the matter that will be learned.

Now we will talk about of four main prerequisite knowledge that will be used in this explanation. They are the prerequisite of learning to make pattern, to describe pattern, to communicate mathematical relation by using equality sign and the last one is to model a problem. The prerequisite knowledge in learning to make pattern in mathematics is computational concept, such as addition, substitution, multiplication, division. It will be begun from learning about simple pattern in the plane (geometrical pattern), continue to more complex geometrical pattern and then learn about simple until complex numerical pattern. The prerequisite knowledge in describing patterns, tables and graphs is introduction of statistics, concept of patterns, tables and graphs, datum/data in single or group form. Teacher holds main role to make student accustomed to understand about data, may be it can be did by asking students to read and analyze data given. The prerequisite knowledge in communicating mathematical relation by using equality sign is fundamental concept of symbols in mathematics, such as adding, substituting, dividing sign, numeric operation, and understanding of the meaning of numbers that located in left or right side of its equality sign. After that students will be thought to complete a numeric operation by its open ended question. The prerequisite knowledge in modeling a problem in mathematics (most of content in mathematics are able to represent by using model), students must have creativity to making another representation, such as using variable to change word problem into mathematical sentences. Then, students choose right strategy to solve the problem by using its model.

For the next research, learning trajectory as a result of this study will be developed as a design learning. The purpose of this design learning is to developing didactical learning to solve algebraic thinking problems, especially for upper grade of elementary school' students.

## 4. CONCLUSION

Algebraic thinking learning in upper elemetary grades is not included in the curriculum, but it is implied in some basic mathematical competencies that use algebraic thinking processes. A study of algebraic thinking skills in elementary schools, especially upper elementary grades, is needed to make the design of learning tailored to the elementary school mathematics curriculum. the study reveals that the average student's ability to think algebra in algebraic thinking is $55.5 \%$. It means that based on the indicators of algebraic thinking, the algebraic thinking ability of upper elementary grades is less good. Also found onbstacles to algebraic thinking that occur in students are obstacles in describing, conveying, and making generalizations about geometric and numerical patterns, and obstacles in showing and analyzing patterns and functions, using words, tables, and graphs, barriers in showing a mathematical relationship by using equal signals, as well as obstacles in modeling the problem situation with objects and using representations, such as graphs, tables, and equations to draw conclusions. Based on the obstacles of thinking thought that have been found, it has been generated learning trajectory to develop algebraic thinking learning design.

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